

## Research Report

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# Developing Content Knowledge for Teaching Assessments for the Measures of Effective Teaching Study

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## RESEARCH REPORT

# Developing Content Knowledge for Teaching Assessments for the Measures of Effective Teaching Study

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This report documents the development of assessments of content knowledge for teaching (CKT) as part of the Measures of Effective Teaching (MET) study, funded by the Bill and Melinda Gates Foundation. The MET study was designed to develop a set of measures that together serve as an accurate indicator of teaching effectiveness. The study was implemented during the 2009–2010 and 2010–2011 school years with more than 3,000 teachers in 6 predominately urban school districts. A total of 5 assessments of CKT were developed, piloted, and then administered as part of the MET study. The CKT assessments focused on the content knowledge used in recognizing, understanding, and responding to the content problems that teachers encounter as they teach a subject. In English language arts (ELA), 2 assessments were developed: 1 for teachers of Grades 4–6 and 1 for Grades 7–9. In mathematics, 3 assessments were developed: 1 for teachers of Grades 4–5, 1 for Grades 6–8, and 1 algebra I. A total of 2,080 final assessments were administered to 1,718 teachers in the 6 participating MET study districts. Assessment results for 194 teachers were excluded due to evidence that assessments were either completed together by 2 or more participants or that insufficient time was devoted to represent a good faith effort at answering the assessment questions. The final sample included 1,886 assessments. Assessment scores included both selected-response and constructed-response (CR) questions. We used information from item level statistics, including percent correct and biserial correlations, to systematically remove poorly performing items in order to improve assessment reliabilities. Item level statistics for each assessment are presented. Descriptive statistics and histograms indicate that participants are well distributed over the range of possible score responses. Assessments had moderate to strong levels of reliability, ranging from 0.69 to 0.83.

**Keywords** Teacher content knowledge; assessment; teaching quality; English language arts; mathematics

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The content knowledge for teaching (CKT) assessments described in this report were developed as part of the larger Measures of Effective Teaching (MET) study. The goal of this development effort was to create a set of assessments that measure the types of content knowledge used in teaching practice and then to examine how these CKT assessments are related to other measures of teaching effectiveness. To provide context for the assessment development, the report starts with a brief overview of the larger MET study and of the general theory of CKT that guided both the development of the assessment framework and assessment forms. The second section of the report describes the process of developing an assessment framework, assessment items, and the piloting and revision steps that led to the construction of final assessments (Appendices A–E). The third section of the report presents results from the administration of the assessments to teachers participating in the MET study and includes information on item performance, assessment scoring and reliability, and the general characteristics of participant scores. Appendix F contains item level statistics (i.e., percent correct and biserial correlations) for all items administered.

## Background

### Measures of Effective Teaching Study

The MET study was designed to develop a set of measures that together serve as an indicator of a teacher's impact on student achievement. The study was implemented during the 2009–2010 and 2010–2011 school years with approximately 3,000 teachers in six predominately urban school districts. To limit the need for extensive additional testing of students, the study focused on grades and subjects in which most states currently test students—Grades 4 through 8 mathematics

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and English language arts (ELA)—plus three additional courses that serve as gateways for high school students—Grade 9 algebra I, English, and biology.

The research study was led by more than a dozen academic institutions, not-for-profit institutions, and other educational consultants. Data about the teachers' practice and their students' achievement were collected from a number of sources, including the following:

- Measures of student achievement gains on existing state assessments and supplemental assessments designed to assess higher-order conceptual understanding.
- Classroom observations, scored using multiple observation protocols, and teacher reflections on their practice.
- Measures of teachers' CKT (mathematics and ELA only).
- Surveys of student perceptions of the classroom instructional environment.
- Surveys of teachers' perceptions of the working conditions and instructional support at their schools.

The measures of teachers' CKT in mathematics and ELA were developed during the first year of the study and were administered to teachers participating in the study during the second year.

## Content Knowledge for Teaching

Content knowledge is a long-established basic prerequisite for teaching a subject, and it is an essential requirement for teacher certification (Hill, 2007). However, scholars have argued that teachers need to develop forms of content knowledge that go beyond basic content proficiency to be effective in the classroom. The idea that teachers need to understand and use content in ways particular to teaching was captured in arguments developed by Shulman, who conceptualized *pedagogical content knowledge* (PCK) as comprising

the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others. . . . Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (Shulman, 1986, p. 9)

The notion of PCK has been influential to the education field. These ideas have contributed to research that has focused on the central role of content in teaching and that has provided a way to conceptualize teaching as professional work with its own unique knowledge base (Ball, Thames, & Phelps, 2008, p. 392).

Ball et al. (2008) proposed a domain argument that includes the full range of content knowledge used in teaching a subject, with knowledge falling under two major categories: content knowledge and PCK. PCK includes three types of content knowledge that are a mix of content knowledge and knowledge of students, teaching, or curriculum. *Knowledge of content and students* includes, for example, knowledge of particular content misconceptions or confusions that students are likely to exhibit. *Knowledge of content and teaching* includes knowledge of the productive ways of representing particular content to meet student learning needs. *Knowledge of content and curriculum* includes knowledge such as how to use existing curricula to teach specific content.

Content knowledge for teaching also includes three categories of content knowledge that do not involve knowledge of students, teaching, or curriculum. *Common content knowledge* is defined by its shared use in common across adult pursuits other than just teaching. For example, engineers and mathematics teachers both use knowledge of how to solve algebraic problems in conducting their day-to-day work. *Horizon content knowledge* is defined by knowledge of how different content ideas are connected across the content domain, often with more elementary or basic ideas connected to more complex or advanced ideas. For example, one would not expect a high school geometry course to include non-Euclidean geometry, but it is helpful for a teacher teaching the triangle sum theorem to know its connection to non-Euclidean geometry, as well as the mathematical importance of that connection. *Specialized content knowledge* includes types of content knowledge that are only used in teaching. For example, while someone skilled in mathematics may use a particular mathematical strategy or method to solve a set of problems, the teacher must understand a range of strategies that could be used to represent the math in different ways to address different types of student learning needs, differences in the mathematics involved in each of the strategies, and whether such strategies are mathematically valid and generalizable.



These are examples of a type of pure mathematics that are only needed for and used in the work of teaching mathematics and that do not require the additional knowledge about students or content representations that comprise PCK.

Emerging theories about what constitutes content knowledge for teaching have received interest from teacher educators, policy makers, and teachers themselves, because these ideas draw a direct connection between the work of teaching a subject and the content knowledge needed in order to carry out this work. While clearly teachers need to understand the content that their students are learning, evidence that teaching this content can require forms of content knowledge that go far beyond more conventional content proficiency strengthens arguments that teaching is professional work with its own unique professional knowledge base. These ideas also point to the potential and need for assessments that capture forms of content knowledge used in teaching that differ from what is measured on conventional knowledge tests.

## Developing a Design Framework and Content Knowledge for Teaching Assessments

### A Design Framework for Content Knowledge for Teaching Assessments

Building on the literature, we define CKT as the content knowledge used in recognizing, understanding, and responding to the content practices that teachers engage in as they teach a subject. While much of the literature on content knowledge for teaching has focused on distinctions in the types of knowledge used in teaching, we place the emphasis instead on teaching itself and seek to identify the main ways that content knowledge is used in teaching practice (Ball & Bass, 2003). This focus on teaching practice is an important component of current theories of CKT. Domains such as PCK and its subcomponents are defined by the ways in which content is encountered and worked within teaching. In the classroom, PCK exists because teachers work with students' content confusions or with content representations in teaching content. It is the work of teaching and how content is encountered in teaching that distinguishes domains of CKT from content as it is used outside of teaching practice. However, in our work to develop measures of CKT, we have shifted the emphasis in both conceptualization and actual question design from distinctions that differentiate knowledge type to distinctions that differentiate how knowledge is used in teaching. Instead of seeking to develop and design items that measure one type of CKT rather than another, we focus on the practices or tasks of teaching that make up the work of teaching subjects such as ELA and mathematics and organize our domains around those tasks. As a consequence, the assessment questions organize around the work of teaching in whatever mix of the various components of CKT that are needed to address the content problems that teachers need to recognize, understand, or respond to in a particular task.

Our perspective in assessing CKT is defined by the recurrent practices that make up the work of teaching subjects such as mathematics and ELA. These recurrent practices, known as *tasks of teaching*, define what knowledge is needed for teaching and, thus, form a design framework for CKT assessments that directly links content knowledge to teaching practice. One could label this a practice-based approach, because it takes instructional practice as the basis for both organizing knowledge and for designing assessment questions that can measure the content knowledge *used in* teaching.<sup>1</sup>

The CKT assessment design framework focuses on the content practices that teachers encounter in the range of settings and roles that define their work. Most obviously, this involves teachers' interactions with students around the content and learning activities that occur within the classroom. However, teachers also engage in many content practices outside of the classroom as they consider or plan how to teach a topic, make sense of student work products to decide what to do next, share teaching with colleagues, design lessons or curriculum materials, interpret standards, prepare students for state tests, and much more. A practice-based framework for assessing CKT must sample from the full range of content practices that define that work of teaching a subject.

These core practices, or *tasks of teaching*, are the organizing foundation for the CKT assessment design framework. Many tasks of teaching are common across subject areas and grade levels and thus, create a common structure that in turn provides a basis for elaborating the tasks of teaching encountered in a specific subject area and grade level. These elaborated descriptions of tasks of teaching provide the basis for defining the content problems that teachers need to recognize, understand, and respond to and that are the target of particular CKT assessment questions.

The design framework developed for the MET study is outlined in Table 1 and describes the content-specific tasks that comprise the work of teaching at three levels: first a general level that is shared across subjects; next, a level of tasks that are specific to defined content domain; and finally, a level that is defined by integration of the actual content topics that are taught to students with the work of teaching that content. At the highest level, represented in the two left-most columns, the content tasks of teaching are general. Tasks of teaching at this level include, for example,

Table 1 Content Knowledge for Teaching (CKT) Assessment Design Framework Tasks of Teaching

Task of teaching	Description	Mathematics	English language arts (ELA)
1. Anticipating student challenges, misconceptions, partial misconceptions, alternate conceptions, strengths, interests, capabilities, and background knowledge	<i>This task of teaching describes the work of integrating knowledge about content to be learned and about students, and anticipating how students are likely to interact with the content and content practices. This work is key in planning instruction, both in advance and in the moment as adjustments are made. These anticipations can become the basis for selecting appropriate explanations, examples, or tasks as instructional strategies.</i>	<ul style="list-style-type: none"> <li>Anticipating student challenges in reasoning about and doing mathematics due to the interplay of content demands and students' understanding</li> <li>Anticipating the impact of limited English language proficiency on students' comprehension of math concepts and on their ability to present mathematical ideas, make mathematical arguments, and give explanations</li> <li>Anticipating likely misconceptions, partial conceptions, and alternate conceptions about particular mathematics content and practices</li> <li>Anticipating student interest and motivation around particular mathematics content and practices</li> </ul>	<ul style="list-style-type: none"> <li>Anticipating student challenges in reading, writing, speaking, or listening due to the interplay of content demands and students' understanding</li> <li>Anticipating the impact of limited English language proficiency on students' comprehension of text and speech and on their written and spoken expression</li> <li>Anticipating likely misconceptions, partial conceptions, and alternate conceptions about particular ELA concepts, texts, resources, and processes</li> <li>Anticipating student interest and motivation around particular ELA concepts, texts, resources, and processes</li> <li>Anticipating how students' background knowledge, life experiences, and cultural background can interact with new ELA concepts, texts, resources, and processes</li> </ul>
2. Evaluating student ideas evident in work, talk, actions, and interactions	<i>This task of teaching describes the work of making sense of things that students do, say, and produce. It can involve deciding if an idea is valid and/or if an explanation is adequate in a particular context, identifying evidence of partial or alternate understandings, or analyzing errors. Evaluating includes characterizing, analyzing, describing, and judging student work.</i>	<ul style="list-style-type: none"> <li>Evaluating student work, talk, or actions in order to identify conceptions in mathematics, including incorrect or partial conceptions</li> <li>Evaluating student explanations or arguments for use of appropriate mathematical practices</li> <li>Evaluating nonstandard responses for evidence of mathematical understanding and in terms of efficiency, validity, and generalizability</li> <li>Evaluating student representations for evidence of mathematical understanding</li> <li>Evaluating or comparing multiple solutions to a mathematics problem or explanations of a mathematical concept or procedure</li> <li>Evaluating discussion among groups of students for evidence of understanding mathematics concepts and practices</li> </ul>	<ul style="list-style-type: none"> <li>Evaluating student work, talk, or actions in order to identify conceptions, including incorrect or partial conceptions, about ELA concepts, texts, and processes</li> <li>Evaluating student work, talk, and actions for evidence of strengths and weaknesses in reading, writing, speaking, and listening</li> <li>Comparing multiple responses to an ELA task</li> <li>Evaluating discussion among groups of students for evidence of understanding ELA concepts, texts, and processes</li> </ul>

Table 1 Continued

Task of teaching	Description	Mathematics	English language arts (ELA)
3. Explaining concepts, procedures, representations, models, examples, definitions, and hypotheses	<i>This task of teaching describes the crafting and using of appropriate explanations at any point in time when explanations are called for, including in setting purposes for instruction, planning, giving feedback, or in responding to students in the moment. Explaining also includes activities that show, such as modeling.</i>	<ul style="list-style-type: none"> <li>Explaining mathematical concepts, or why a mathematical idea is true</li> <li>Explaining mathematical procedures</li> <li>Explaining mathematical representations</li> <li>Explaining mathematical models</li> <li>Explaining mathematical examples</li> <li>Explaining mathematical definitions</li> <li>Explaining mathematical hypotheses</li> <li>Explaining why a particular definition, model, or representation serves a particular purpose</li> <li>Interpreting a particular representation in multiple ways to further understanding</li> <li>Explaining why a practice, process, or procedure is efficient or appropriate</li> <li>Explaining the context of a mathematical conjecture and why it is a conjecture</li> </ul>	<ul style="list-style-type: none"> <li>Explaining literary or language concepts, using definitions, examples, and analogies when appropriate</li> <li>Explaining processes of reading, including why certain processes are appropriate for particular texts and/or tasks</li> <li>Explaining processes of writing, including why certain processes are appropriate for particular tasks</li> <li>Explaining processes of listening and speaking, including why certain processes are appropriate for particular topics, groups, and/or task</li> <li>Explaining the use of representations to support understanding or development of text</li> <li>Explaining hypotheses about texts</li> </ul>
4. Creating and adapting resources for instruction (examples, models, representations, explanations, definitions, hypotheses, procedures)	<i>This task of teaching describes the work of inventing new instructional tools or adapting available ones to fit particular purposes. While curricular resources can be a source of material for adaptation, student-generated materials can also be adapted into opportunities for learning. The creation or adaptation of instructional tools is thought of as having an instructional goal, examples of which are in the subject-specific bulleted lists. Examples are selected for a reason—to introduce new material, to showcase certain attributes of content, or to challenge students' current conceptions.</i>	<ul style="list-style-type: none"> <li>Creating and adapting examples to introduce a concept or illustrate an idea or to demonstrate a strategy, procedure, or practice</li> <li>Creating and adapting examples that support particular mathematical strategies or to address particular student questions, misconceptions, or challenges with content</li> <li>Creating and adapting representations or models to support students' mathematical understanding</li> <li>Creating and adapting multiple representations or models to support students' mathematical understanding across representations</li> <li>Creating and adapting representations or models that support multiple interpretations</li> <li>Creating and adapting definitions to fit instructional purposes</li> <li>Adapting student-generated conjectures to support instructional purposes</li> <li>Creating and adapting procedures for working with content</li> </ul>	<ul style="list-style-type: none"> <li>Creating and adapting examples or model texts to introduce a concept or to demonstrate a literary technique or a reading, writing, or speaking strategy</li> <li>Creating and adapting examples or model texts to develop understanding of a concept, literary technique, or literacy strategy, or to address particular student questions, misconceptions, or challenges</li> <li>Creating and adapting representations (e.g., graphic organizers, think-alouds) to support understanding or development of text or other products</li> <li>Creating and adapting definitions of ELA concepts and processes to fit instructional purposes</li> <li>Adapting student contributions to fit instructional purposes</li> <li>Creating and adapting procedures for writing, speaking, listening, or working with texts</li> <li>Creating and adapting analogies to support student understanding of ELA concepts, texts, and processes</li> </ul>

Table 1 Continued

Task of teaching	Description	Mathematics	English language arts (ELA)
5. Evaluating and selecting resources for instruction (examples, models, representations, explanations, definitions, hypotheses, procedures)	<i>This task of teaching describes the work of evaluating instructional resources and selecting those most appropriate for a particular instructional purpose. This work can occur through advance planning or during instruction. Teachers may evaluate and select from curricular materials or from potential resources generated during the course of instruction, including work generated by students.</i>	<ul style="list-style-type: none"> <li>Evaluating and selecting examples to introduce a concept or illustrate an idea or to demonstrate a strategy, procedure, or practice</li> <li>Evaluating and selecting examples that support particular mathematical strategies or to address particular student questions, misconceptions, or challenges with content</li> <li>Evaluating and selecting representations or models to support students' mathematical understanding, or in terms of validity, generalizability, or fit to the concept, calculation, etc. to be represented</li> <li>Evaluating and selecting representations or models that support multiple interpretations</li> <li>Evaluating and selecting definitions to fit instructional purposes</li> <li>Evaluating and selecting explanations of mathematical concepts for potential to support mathematical learning or in terms of validity, generalizability, or explanatory power</li> <li>Evaluating and selecting procedures for working with mathematics content</li> <li>Evaluating and selecting student-generated conjectures to support instructional purposes</li> <li>Evaluating and selecting resources for their potential to support mathematical learning</li> </ul>	<ul style="list-style-type: none"> <li>Evaluating and selecting examples or model texts to introduce a concept or to demonstrate a literary technique or literary strategy</li> <li>Evaluating and selecting examples to develop understanding of a concept, literary technique, or literary strategy, or to address particular student questions, misconceptions, or challenges with content</li> <li>Evaluating and selecting representations (e.g., graphic organizers, think-alouds) to support understanding or development of text or other products or in terms of fit to the concept to be represented</li> <li>Evaluating and selecting definitions of ELA concepts and processes to fit instructional purposes</li> <li>Evaluating and selecting explanations of ELA concepts, texts, and processes for their potential to support learning</li> <li>Evaluating and selecting procedures for writing or working with text</li> <li>Evaluating and selecting resources to support particular instructional goals</li> <li>Evaluating and selecting analogies to support student understanding of ELA concepts, texts, and processes</li> </ul>
6. Developing questions, activities, tasks, and problems to elicit student thinking	<i>This task of teaching includes the work of developing and using questions, tasks, and problems to engage students in developing content understanding. The development of questions, tasks, and problems is used to make student understanding visible and to build deeper and more accurate understanding.</i>	<ul style="list-style-type: none"> <li>Creating or adapting problems or questions with the potential to elicit student mathematical thinking, discussions, justifications, or explanations</li> <li>Creating or adapting problems that support particular mathematical strategies and practices</li> <li>Creating or adapting questions, activities, tasks, or problems that demonstrate desired mathematical characteristics</li> <li>Creating or adapting classes of problems that address the same mathematical concept or that systematically vary in difficulty and complexity</li> <li>Creating or adapting questions, activities, or tasks to elicit evidence that students have a particular mathematical understanding or skill</li> </ul>	<ul style="list-style-type: none"> <li>Creating or adapting questions with the potential to elicit discussion or analysis of a text or a literary concept</li> <li>Creating or adapting prompts or questions with the potential to elicit productive student writing</li> <li>Creating or adapting activities or tasks that support the development of a particular literary understanding or skill</li> <li>Creating or adapting questions, activities, or tasks to elicit evidence that students have a particular literary understanding or skill</li> </ul>

Table 1 Continued

Task of teaching	Description	Mathematics	English language arts (ELA)
7. Evaluating and selecting student tasks (questions, tasks, problems) to elicit student thinking	<i>This task of teaching includes the work of evaluating and selecting things for students to work on or interact with. The evaluation and selection of student tasks is thought of as having an implicit instructional goal and relies on anticipations about how students will understand and interact with the tasks.</i>	<ul style="list-style-type: none"> <li>• Evaluating and selecting problems or questions to elicit student mathematical thinking, justifications, or explanations</li> <li>• Evaluating and selecting problems that support particular mathematical strategies and practices</li> <li>• Evaluating and selecting questions, tasks, or problems that demonstrate desired characteristics</li> <li>• Evaluating and selecting classes of problems that address the same mathematical concepts or that systematically vary in difficulty or complexity</li> <li>• Evaluating and selecting questions, activities, or tasks to elicit evidence that students have a particular mathematical understanding or skill</li> </ul>	<ul style="list-style-type: none"> <li>• Evaluating and selecting questions, activities, or tasks to elicit discussion about a specific text or literary concept</li> <li>• Evaluating and selecting questions to elicit productive student writing</li> <li>• Evaluating and selecting questions, activities, or tasks to support the development of a particular literary understanding or skill</li> <li>• Evaluating and selecting questions, activities, or tasks to elicit evidence that students have a particular literary understanding or skill</li> </ul>
8. Doing the work of the student curriculum	<i>This task of teaching includes doing the student level tasks that make up the intended curriculum. Doing the student level work is not uniquely demanded by teaching, but is an essential part of the content work necessary to do in the course of teaching, especially in preparation for assigning student tasks. This work is often incorporated into or prerequisite for doing other tasks, such as anticipating student difficulties or selecting problems or tasks.</i>	<ul style="list-style-type: none"> <li>• Doing the work that will be demanded of the students as part of the intended curriculum</li> </ul>	<ul style="list-style-type: none"> <li>• Doing the work that will be demanded of the students as part of the intended curriculum</li> </ul>



anticipating student difficulties, evaluating student work, and explaining concepts. At this level, tasks of teaching are applicable to all subjects regardless of topic or grade level and, thus, provide a general organization that can guide development for any assessment of CKT. Tasks of teaching that are specific to a subject are indicated in the final two columns.

The third level of the hierarchy specifies the content topic level distinctions that are the focus of each of the tasks of teaching. For example, a task of teaching, such as anticipating student challenges, misconceptions, partial misconceptions, alternate conceptions, strengths, interests, capabilities, and background knowledge (see Table 1), might focus on anticipating likely student misconceptions around number concepts, rational numbers, proportional reasoning, algebra, and so forth. The third level of the framework represents the particular ways that teachers work with the topical content that they are responsible for teaching. We have not attempted to represent this third level of the framework in Table 1 because of the level of detail that would be involved. In an ideal form of test development, all of the ways that content intersects with tasks of teaching would be identified and there would be a clear mapping between this elaborated map of content teaching and the test items that sample from this domain. This level of domain development was not undertaken for the MET assessments. However, the actual items, which are presented in the appendices, provide a useful illustration of how tasks of teaching involve work with particular topical content and the range of topical content that was addressed at the third level of the framework.

The CKT assessments, and in particular the third level of the CKT framework, are guided by attention to general topic level strands that define the content that is taught and learned at the grade levels that are assessed. As a practical matter, identifying these general topic strands can present challenges in developing a valid and relevant assessment. Often states and even districts have quite different standards for the student content that is taught, the grade level at which this content is taught, and to what level of content proficiency. These differences make it difficult to justify the specific content that should be emphasized in a CKT assessment. It is worth noting that recent changes in student standards toward a more commonly shared set of student standards (e.g., the Common Core State Standards [CCSS]) could have a major impact on the development of CKT assessments of mathematics and ELA. If, for example, CCSS were fully adopted by all schools and states in the United States, then the CCSS would become the basis for the third level of the hierarchy. At this point, however, no such shared basis exists, and the third level of the hierarchy is by necessity a set of decisions and compromises in emphasizing content that is widely shared enough to be deemed a necessary component of CKT for a particular subject and grade span.

The list of tasks of teaching (see Table 1) is not taken to be comprehensive, although we have made efforts to generalize across content domains. This list is not intended to imply that this is all teaching consists of, but it is a result of our focus on the *content* problems teachers must contend with in the work of teaching. The tasks on the list are not intended to be seen as entirely exclusive of one another. In many cases, the work of teaching might include multiple tasks, or a single task might potentially be described in more than one way. For example, a piece of student work completed in class might first be evaluated by the teacher, then become an example that is evaluated and selected because it highlights a particular point that the teacher has in mind due to anticipating certain misconceptions about the content. Larger tasks of teaching not represented here may be made up of smaller tasks of teaching. Responding to students, for example, is a combination of evaluating what the student has said and done, choosing an appropriate response, and then delivering that response. The tasks of teaching listed here would include evaluation of the student work and creation of an explanation, example, model, representation, or something else as appropriate to the student understandings or misunderstandings at hand.

Tasks of teaching are the organizing basis for the design framework because each task represents a natural unit of teaching practice that has been identified in research or other relevant analyses of teaching practice as being a critical part of teaching. As natural units of practice, they often represent the coordination of different knowledge domains of CKT. For example, a task of teaching could simultaneously draw on common content knowledge, specialized content knowledge, and knowledge of content and teaching. Rather than artificially segmenting instructional problems to align with a discrete knowledge domain of CKT, the tasks of teaching attempt to capture how teachers actually work with content in their day-to-day and moment-to-moment practice. This supports validity claims that assessments built on this framework measure the intended knowledge and reasoning and also increase the face validity of assessment tasks by situating them in contexts that are perceived by teachers as authentic and relevant.

**Table 2** Advisory Group Members

Consultant	Affiliation	Content area
Margarita Calderón	Johns Hopkins University	English language arts
Pam Grossman	Stanford University	English language arts
Carol Lee	Northwestern University	English language arts
James Marshall	University of Georgia	English language arts
Deborah McCutchen	University of Washington	English language arts
P. David Pearson	University of California, Berkeley	English language arts
Deborah Ball	University of Michigan	Mathematics
William Bush	University of Louisville	Mathematics
Heather Hill	Harvard University	Mathematics
Erik Jacobson	University of Georgia	Mathematics
Rebecca McGraw	University of Arizona	Mathematics
Judit Moschkovich	University of California, Santa Cruz	Mathematics
Denise Spangler	University of Georgia	Mathematics
Jon Star	Harvard University	Mathematics

## Assessment Development

### Development Teams

The practice-based approach that led to the definition of the CKT construct was also the basis for developing assessments of CKT. For each subject area, a development team was created to generate items that were directly linked to the framework and, thus, to teaching that subject. Each development team comprised a mix of individuals, including professional assessment developers at Educational Testing Service (ETS), researchers working on the CKT project, teacher educators, and educators working directly with K-12 students. The goal of item development was to create assessment tasks that elicited the knowledge and reasoning used to recognize, understand, and act on content problems encountered in these tasks of teaching.

Advisory groups were recruited to provide consultation and advice during the framework and item development. Members of the advisory committees (Table 2) were nationally recognized scholars in the field of mathematics or ELA teacher education. These consultants provided feedback and guidance during the early development of the CKT task of teaching framework. They were also given the opportunity to review and provide feedback on all of the items that were piloted in 2010.

The development of the teacher knowledge measures was a joint effort of researchers from the University of Michigan (UM) and researchers and assessment specialists from ETS and was built directly on the extensive experience gained from the Learning Mathematics for Teaching (LMT) project at the UM. The proposed plan was to develop a series of six assessments of teacher knowledge for teachers of the following subjects and grades:

- Mathematics Grades 4–5
- Mathematics Grades 6–8
- Algebra I
- ELA Grades 4–5
- ELA Grades 6–8
- English Grade 9

However, a series of discussions among the research team and advisors led us to decide to limit development to two ELA assessments, as the natural break was one in which the respective curricula emphasized skill development versus literature. Therefore, two ELA teacher knowledge measures were developed: one for teachers of ELA in Grades 4–6 and the second for teachers of ELA in the Grades 7–9. Three mathematics assessments were developed, as described above. During the project proposal phase, consideration was also given to developing a measure of content knowledge for teaching high school biology, but after discussions with the MET leadership, this was not deemed to be feasible given the constraints of the project budget, the short development timeline, and limited expertise in this area.

The assessments of teacher knowledge were designed to be delivered over the Internet, allowing teachers to take assessments when they chose. From the beginning, we recognized the challenge of finding a reasonable balance between



Ms. Hupman is teaching an introductory lesson on exponents. She wants to give her students a quick problem at the end of class to check their proficiency in evaluating simple exponential expressions. Of the following expressions, which would be least useful in assessing student proficiency in evaluating simple exponential expressions?

- A)  $3^3$
- B)  $2^3$
- C)  $2^2$
- D) All of these are equally useful in assessing student proficiency in evaluating simple exponential expressions.

**Figure 1** Sample mathematics content knowledge for teaching (CKT) item.

Ms. Rice begins a unit on memoir writing by reading a passage from a literary model. She then asks students to complete a warm-up activity to help them generate ideas for their own writing.

For each assignment, indicate whether or not it will help students focus their brainstorming on generating a memoir.

	Will help focus brainstorming	Will not help focus brainstorming
(A) Write a poem about the ways you have changed, using the form "I used to be...but now I am..."		
(B) Write a sequence of sentences describing some of your experiences, beginning each sentence with the phrase "I remember."		
(C) Write a few adjectives that describe your personality.		
(D) Write down some of your favorite foods and describe what you like about them.		

**Figure 2** Sample English language arts (ELA) content knowledge for teaching (CKT) table item.

assessment length, participant burden, and adequate assessment reliabilities. Given that there were no stakes associated with assessments in the context of the MET study and participation was voluntary, the assessment length was largely driven by a decision to keep the assessments relatively short to reduce participant burden. If these teacher knowledge assessments had been intended to be used to discriminate among teachers for the purpose of high stakes decisions such as tenure and firing, then very high levels of assessment reliability and associated precision for individual classifications would be necessary. Achieving high levels of reliability is typically accomplished by creating long assessments. However, reliabilities can also be enhanced by making use of performance data from pilot administrations to select high performing items and strike a balance in design that maximizes assessment reliabilities and minimizes participant burden. For these particular assessments, our goal was to create assessments that had reliabilities above 0.70 and could be completed in an hour or less.

### Question Types

The question development work built directly on the experiences of the LMT project at the UM. The LMT measures comprise two types of questions: single-selection multiple-choice, in which the single best answer is chosen from among the four answer choices given, and what we refer to as *table questions*, in which a common stimulus is presented with two or more answer choices (presented in columns), and a separate response must be given for each scenario presented in the rows of the table. Typically, table items present a yes or no response option. Likewise, the majority of the questions included in the MET assessments of teacher knowledge used these same two response types. Sample MET questions are shown in Figures 1 (example of a single-selection multiple-choice question) and 2 (example of a table question).

For the purposes of clarity, in all subsequent descriptions of the assessments and in the discussion of the data analyses, each response to a selected-response question will be referred to as an item. Thus, as can be seen in Figures 1 and 2, a single-selection multiple-choice question corresponds to a single item, but a table question corresponds to several items, one for each row in the table (e.g., the sample table question corresponds to four items). Each table question in the MET teacher knowledge assessments corresponds to three to six items. (A discussion about the independence of the items within a table question is included in the scoring results section below.)

In addition to the two types of selected-response questions described previously, four of the five measures of teacher knowledge (i.e., all except algebra I) also included a small number of open-ended or constructed-response (CR) questions. One compelling reason to include CR questions in assessments like these is that they can capture evidence of certain types of knowledge (e.g., generative reasoning) that is not available from selected-response questions. Although many large-scale assessments (teacher and student) have not included CR tasks because of the significant costs associated with scoring the tasks with human raters, recent developments in natural language processing have made it possible to conceive of scoring short constructed responses using automated scoring technologies. Thus, the proposal for the development of teacher knowledge measures included small-scale efforts to (a) develop CR questions that reliably measure some aspect of the CKT construct and (b) investigate the feasibility of using automated scoring technologies to score the responses. (Since the algebra I assessment was only administered to teachers at one grade level, Grade 9, the expected sample size was too small to support building automated scoring models, and thus, no CR items were included in the algebra I assessment.)

### ***Item-Writing Procedures***

This assessment development was not a standard effort in two important ways. First, for this study, the assessment development teams were simultaneously working within and iteratively developing a theoretical framework for the assessments. In more traditional assessment development, the assessment design and specifications emerge from a consensus-building process among content-expert development committee members. These specifications often take the form of lists of important content topics, rather than an overarching framework, and are defined prior to actual assessment development work. Second, because the content domains for basic mathematics knowledge and reading ability are fairly well specified, development of tests of teacher knowledge have traditionally followed a rather straightforward procedure, creating questions that focus on content topics that include, and to some degree encompass, the student curriculum for the grade span. This requires assessment developers to have knowledge of their subject area and expertise in constructing assessments but not necessarily deep understanding of teaching. However, the starting point for development of the MET CKT assessments was teaching practice itself, as defined by the tasks of teaching. Therefore, the assessment development required simultaneous attention to teaching practice, teacher content knowledge as it is used in teaching practice, and more standard conceptions of the content knowledge represented by the student curriculum. Not only does this complicate the work because additional, deeper knowledge is required, but it also requires conventionally trained assessment developers to learn how to attend to dimensions of teaching in assessment development.

We found that to do this work, assessment development teams needed a solid grounding in instruction in their subject matter area, and both needed to develop the skills to examine particular tasks of teaching and to identify the knowledge entailed by those tasks. Consequently, the mathematics and ELA assessment development teams each comprised researchers whose work focused on the study of teaching, subject matter experts, teacher educators, and assessment specialists, many who also had teaching experience. The development teams also collaborated with external groups with specific expertise around teaching, such as a group of professional writers who both work directly with students and provide professional development to teachers of creative writing.

Both of the development teams met regularly, virtually, and face to face, throughout the first year of the study to develop a pool of questions that could be used in piloting and as examples to generate additional questions. Members of the development teams drafted questions that focused on the tasks of teaching and core content topics for each particular assessment. All questions were reviewed and discussed in detail multiple times by the development teams to ensure that the questions would elicit the desired evidence of teacher knowledge and to inform the continuous development of the CKT framework. Questions that were deemed acceptable by the development teams were then reviewed for compliance with editorial standards and best practices for assessment development. All revisions to the questions were documented and tracked to enable us to study the development process at a future time.

### ***Pilot***

A pool of more than 200 questions was developed across the five measures. During the summer of 2010, two forms of each of the five assessments were assembled and piloted with a convenience sample of practicing teachers, teaching the

**Table 3** Summary of Question Types, Item Exclusion, and Assessment Reliability

Assessment form	Total SR items	SR items excluded	SR item reliability	Total CR items	Final reliability
Mathematics 4–5	38	0	0.74	2	0.76
Mathematics 6–8	46	0	0.82	2	0.83
Algebra I	37	2	0.77	0	0.77
ELA 4–6	51	5	0.71	2	0.74
ELA 7–9	48	3	0.66	2	0.69

*Note.* SR = selected response; CR = constructed response; ELA = English language arts.

appropriate subject area and grade level. Extensive efforts were made to recruit potential participants for the pilot administration of the MET CKT items. More than 13,000 teachers were sent invitations to participate. Potential participants were recruited in numerous ways, including direct individual contact using information provided by supporting school districts; charter school operators; and relevant organization LISTSERVs, newsletters, and Web sites. Approximately 1,600 teachers consented to participate in the pilot study, with a total of 1,751 assessments completed. Those teaching at the elementary level were given the option to take both mathematics and ELA assessments.

Participants in the MET study were not allowed to take the pilot assessments. The pilot assessments, along with a short demographic survey, were administered using the same Web-based assessment delivery platform that was used for the MET study administrations. Each assessment was designed to take approximately 1 hour to complete and was administered to 150–200 participants. Detailed analyses were then conducted of the item performance to examine item difficulty, discrimination, and latency.

In parallel with the piloting, a small number of cognitive interviews were conducted with teachers (mostly from New Jersey), using a subset of the questions. These interviews were designed to elicit teachers' thinking as they worked through the questions and to provide information about potential sources of confusion and/or construct-irrelevant variance. Interviews were used to show whether and how effectively the intended constructs were actually being measured by the assessment questions. The results of these interviews were used to support the revision and refinement of the CKT questions included in the assessment forms for the study administration.

Also in parallel with the pilot administration, all items were provided to an external advisory committee for review and comment.

Data from the psychometric analyses and cognitive interviews, along with comments from the external reviewers, were used by the development teams to select and revise the questions for inclusion in the assessments administered to the MET study participants. The assessments as administered are summarized in Table 3, with the actual questions administered on each form presented in Appendices A–E.

### **Final Assessments**

Assessment reliability (Cronbach's raw alpha) ranged from 0.69 to 0.83 for the CKT assessments based on selected-response and CR items (Table 3). Certain selected-response items were excluded from scoring and item analysis through a process elaborated in the Item Analysis and Assessment Reliability section of this report.

A number of questions appeared on multiple assessments. Eight questions appeared on both ELA assessments. Six questions were common to the mathematics 4–5 and mathematics 6–8 assessments. Four questions appeared on both the mathematics 6–8 and algebra I assessments. One question was common to the mathematics 4–5 and algebra I assessments. As discussed in the results, these common items provided an opportunity to investigate differences in item performance across assessments and groups of teachers.

### **Administration and Sample**

The administration of MET assessments of teacher knowledge occurred during the 2010–2011 school year. We administered 2,080 CKT assessments to teachers among six districts participating in the MET study. Teachers received unique URLs to access and complete assessments online at any time during the administration period. The assessment for teachers of mathematics in Grades 4 and 5 was administered in fall of 2010, and the other four assessments were administered in early 2011. Assessments were administered to teachers at the MET-specified grade level (i.e., Grade 4 and Grade 5

**Table 4** Exclusion Codes by Assessment

	Assessment					
	Math 4–5 <i>n</i> = 465 <i>n</i> (%)	Math 6–8 <i>n</i> = 398 <i>n</i> (%)	Algebra I <i>n</i> = 148 <i>n</i> (%)	ELA 4–6 <i>n</i> = 635 <i>n</i> (%)	ELA 7–9 <i>n</i> = 434 <i>n</i> (%)	Total <i>n</i> = 2,080 <i>n</i> (%)
Exclusion category						
Missing responses	0 (0)	3 (1)	0	2 (<1)	4 (1)	9 (<1)
Latency/response pattern	8 (2)	5 (1)	3 (2)	13 (2)	4 (1)	33 (2)
Unusual agreement	60 (13)	17 (4)	2 (1)	65 (10)	8 (2)	152 (7)
Final scored sample	397 (85)	373 (94)	143 (97)	555 (87)	418 (96)	1,886 (91)

*Note.* ELA = English language arts. Percentages shown are with respect to the assessment total *n*.

mathematics teachers received the mathematics 4–5 assessment). The algebra I assessment was administered only to Grade 9 algebra I teachers.

### Excluding Cases From the Scored Sample

We reviewed the assessment results in order to identify those that could not be used to generate reliable scores (e.g., cases of incomplete assessments in which responses were given only to the first few questions or cases in which the entire assessment was completed within a few minutes). In addition, as part of the review process, we noticed patterns in the timing data that suggested possible problems, including collaboration among groups (either pairs or larger groups) of test takers. We then conducted a systematic and more detailed analysis of assessment data. As a result, we excluded 194 assessment results from the final sample. These cases were not included in item analyses and were not individually scored. Cases were excluded for a variety of reasons, but the largest number of exclusions were cases of unusual agreement, instances in which two or more teachers appearing to have collaborated in ways that rendered it impossible to attribute the scores to a particular individual. We excluded additional cases for other reasons that created concern about good faith effort, including questionable response patterns and response times that were not sufficiently long. The excluded cases are summarized in Table 4.

The following descriptions address each of the rules that we applied for this set of analyses. While the overall exclusion rate is approximately 9%, exclusions were more frequent for elementary level assessments.

#### Missing Responses

In these cases, the teacher submitted fewer than 75% of required item responses to complete the assessment (i.e., 25% or more of that teacher's responses were missing). In nearly all cases, these were sequential responses at the end of the assessment, as teachers were required to enter a response in order to proceed. Thus, these individuals exited the assessment prior to completion.

#### Latency and Response Pattern

Teachers completed the assessment at a rate deemed to be too fast to have considered items and thought about response choices. Assessments for which the total latency was less than 5 minutes or for which either half of the assessment was completed in less than 2 minutes and 30 seconds were excluded. Time spent on CR items was not included in this criterion.

We checked each teacher's set of responses for predominant submission of any single response option that suggested that items were not given serious consideration. If over 80% of overall responses or the entire second half of an assessment were selected as a single response option (e.g., the second choice for each item was selected), then the assessment results were flagged. Latency data were used as confirmatory evidence for these types of response patterns. All of these cases were also flagged for speedy assessment completion.

#### Unusual Agreement

We used statistical analysis to compare all possible pairings of teachers in the entire sample to detect unusual agreement. The methodology for detecting unusual agreement was developed at ETS as a means of test security; the numerical value

generated is known as the K-index (Holland, 1996; Lewis & Thayer, 1998). The software matched all possible pairs of teachers' incorrect response choices on multiple-choice items. The K-index is

an estimate of the chance that at least the observed number of matching incorrect responses would occur if the two test takers being compared were working independently on the section of the test being analyzed. If this probability is sufficiently small, it is taken as an indicator of "unusual agreement" between the two sets of incorrect responses being compared. (Lewis & Thayer, 1998, p. 1)

Other characteristics of this detection method include:

- Matched correct responses are not included in these analyses because they may indicate that respondents independently know the content and, thus, provide weak evidence of unusual agreement.
- Omitted responses are ambiguous and are not considered evidence for unusual agreement.
- In order to be able to conduct these analyses, there is a minimum number of multiple-choice items for which response data are available (each of the five CKT assessments met this minimum threshold).
- Item difficulty is not controlled for in this analysis.

For this analysis, if the K-index for a pair of assessment results was less than .01, then those results were flagged for unusual agreement. As described below, flagging by itself was not sufficient to exclude a score. Given a large sample size and all possible pairs of teachers for each assessment, many pairs would be identified by chance. Therefore, we collected further evidence to confirm unusual agreement between teachers, identified in pairs, whose results had a K-index below .01.

- A. Same district: The teachers in a pair are in the same district, indicating they have the opportunity to complete the assessment together.
- B. Same school: The teachers in a pair are in the same school, indicating they have the opportunity to complete the assessment together.
- C. End time: The teachers in a pair completed the assessment within 10 minutes of each other.
- D. Similar answers on multiple-choice questions: Either teacher in an identified pair must not differ in more than two incorrect responses for multiple-choice questions.
- E. Similar answers on table questions: The K-index is determined only using the multiple-choice items. In this analysis, we also checked the consistency of responses on the table questions. Teachers in an identified pairing must not differ in more than two incorrect table item responses. This criterion provided confirmatory evidence of unusual agreement.

In summary, if the K-index is below .01 and criteria A and one of B, C, D, or E are satisfied, we excluded that case on the basis of unusual agreement.

Recognizing that the K-index analysis has weak power when applied to pairs of test takers with few incorrect items, we also reviewed all of the assessment results for teachers in the same schools. We compared their assessment end times, scores, and latencies for the entire assessment and scores and latencies for each half of the assessment. Four pairs of teachers who completed ELA assessments were flagged for completing the assessment at the same time (within 5 minutes of each other) with matching responses and amounts of time spent per item. These teachers were not identified by the K-index because each teacher had only one incorrect response to selected-response items. On the basis of these other data, these were judged to be four cases of unusual agreement, and the eight associated assessment results were excluded from the final analyses.

In sum, 152 assessments were excluded based on the unusual agreement criterion, and the remaining 42 cases due to other criteria used to identify scores that were deemed to provide invalid information on the scored responses.

### Final Data Used for Scoring

The final sample that could be used for scoring contains 1,613 individual teachers who completed 1,886 assessments (273 teachers completed both a mathematics and an ELA assessment). Table 5 summarizes the administration by assessment and by district, respectively.

**Table 5** Final Sample of Content Knowledge for Teaching (CKT) Assessments by District

District	Assessment					Total <i>n</i> (%)
	Math 4–5 <i>n</i> (%)	Math 6–8 <i>n</i> (%)	Algebra I <i>n</i> (%)	ELA 4–6 <i>n</i> (%)	ELA 7–9 <i>n</i> (%)	
A	104 (26)	58 (16)	18 (13)	131 (24)	66 (16)	377 (20)
B	95 (24)	83 (22)	59 (41)	125 (23)	119 (28)	481 (25)
C	85 (21)	113 (30)	25 (17)	141 (25)	120 (29)	484 (26)
D	19 (5)	0 <sup>a</sup>	21 (15)	13 (2)	21 (5)	74 (4)
E	94 (24)	29 (8)	20 (14)	116 (21)	18 (4)	277 (15)
F	0 <sup>a</sup>	90 (24)	0 <sup>a</sup>	29 (5)	74 (18)	193 (10)
Total	397	373	143	555	418	1,886

Note. ELA = English language arts.

<sup>a</sup>Assessment not administered in this district.

## Measures of Effective Teaching Content Knowledge for Teachers Assessments Results

### Scoring Results

We investigated different scoring models and weightings for some of the item types used in the CKT assessments. In this section, we discuss the considerations and decisions made and then present the final score distributions for each of the assessments.

### Selected-Response Items

Two types of selected-response questions were used in the CKT assessments: multiple-choice questions and table questions. Each multiple-choice question was given equal weight and counted as 1 point because each required a single selected response.

We investigated various models of scoring the multiple-response table questions. We examined intra-item correlations (between the items within each table question) and found, for nearly all table questions, that items within a table question were not highly correlated with each other; that is, each item appeared to measure a discrete aspect of CKT. We concluded that each item (within a table question) should be counted as 1 point, as there was no evidence that item performance violated assumptions of local independence. Had there been evidence supporting a lack of local independence, we would have adopted an alternate weighting strategy so as not to inflate reliability estimates artificially. Each table question counted as 3–6 points, depending on the number of items within the table question.

Before calculating final scores for selected-response items, we analyzed item level data (biserial correlations) and assessment reliability. We systematically removed weakly discriminating selected-response items from each assessment before calculating final scores. We describe the item exclusion process in more detail below.

### Constructed-Response Items

To complete the analysis of the assessments of teacher knowledge, we calculated scores for CR items. For each item, we defined a set of concepts or propositions that represented a correct and complete response to the prompt. We then developed a set of scoring rules that matched the presence of some subsets of concepts with a score on the item. For example, if the correct and complete answer for a problem required the presence of concepts A, B, and C, then scoring rules were created to assign a score when A, B, and C were present (i.e., the highest score possible for the item), as well as scores for subsets of concepts in the response (e.g., A and B or B only). Human scorers had to evaluate whether each of the key concepts was present in the response.

Similarly, the automated scoring engine determined the presence or absence of evidence of each of the scoring concepts by determining the extent to which the test-taker response had the same semantic meaning as any or all of the targeted concepts and then applied the respective scoring rules. A large number of prescored responses were used to build models of acceptable and unacceptable responses, and then the computer system used natural language processing techniques to interpret whether different phrasing had the same semantic meaning as a target concept. Varied degrees of success



**Table 6** Summary of Constructed-Response (CR) Items

Assessment	Item sequence number	Number of concepts	Initial score range	Quadratic weighted kappa	Mean scale score <sup>b</sup>
Mathematics 4–5	20	5	0–2	n/a <sup>a</sup>	0.77
Mathematics 4–5	21	2	0–2	0.93	0.99
Mathematics 6–8	22	5	0–4	0.87	1.88
Mathematics 6–8	23	6	0–2	0.77	1.51
ELA 4–6	18	4	0–3	0.68	1.13
ELA 4–6	22	14	0–4	0.83	1.47
ELA 7–9	12	4	0–2	0.87	1.31
ELA 7–9	18	5	0–3	0.70	1.19

*Note.* ELA = English language arts.

<sup>a</sup>Item machine scored only, so a measure of interrater reliability (quadratic weighted kappa) cannot be computed.

<sup>b</sup>Initial score range used to generate raw score by two human raters. An average of the two rater scores was then scaled between 0 and 3 points to calculate a final score for constructed-response items.

were achieved with the automated scoring of the CR items, with the general conclusion that it is possible to develop CR items that measure some aspects of CKT and can be reliably scored using automated scoring techniques. Significant additional research and development around these techniques and the applicability to measures of CKT is needed. With one exception, the reported data are based only on human scores (each response scored by two humans). The scoring of one mathematics item was based entirely on numerical criteria (no text or language evaluation) and was machine scored. The interrater reliability of human scorers was calculated using the quadratic weighted kappa. The reliability of the human scoring for all CR items was judged to be adequate for the purposes of this study (Table 6).

We compared two scaling approaches of CR items to determine the degree to which they factor into final total scores. The first approach was to scale CR items at equal value with selected-response items. The second approach was to scale CR items on a 0- to 3-point scale, which is triple the value of selected-response items. We calculated correlations for each approach with total scores including selected-response items only. Correlations were 0.99 or greater using the single-point scaling approach and 0.97 or greater using the triple-point scaling approach. Factoring this result in with the amount of effort required to answer CR items, we decided to use the 0- to 3-point scale to calculate final scores.

A summary of CR item scoring is provided in Table 6.

## Item Analysis and Assessment Reliability

### Item Level Analyses

We applied the item level scoring methods for each item type to create total raw scores. For each item, we calculated percent of responses correct (i.e., p-plus value) and biserial correlations as measures of item performance. We investigated the impact on the reliability of each assessment when excluding items with poor discrimination, as measured by item biserial correlations (i.e., the relationship between respondents' performance on one item and performance on the entire assessment). First, items, including table items, with negative biserial correlations were excluded. Second, items were eliminated one at a time, starting with the item with the lowest biserial correlation. If the assessment reliability (measured by Cronbach's raw alpha) increased by 0.01 or greater when an item was excluded, then that item was removed from the subsequent score computations, and the process was repeated. If the alpha did not increase by 0.01 or more from the previous value following the exclusion of an item, then that item was not removed from the subsequent calculations, and the process of excluding items from that assessment was concluded. The number of excluded items by form is reported in Table 3. More detailed information about each item is noted in the tables in Appendix F.

### Including Constructed-Response Items in Scale Scores

We investigated the impact on the reliability of each assessment of including CR items, as measured by Cronbach's raw alpha. Assessment reliabilities improved by 0.01 or greater after including CR items. There was no change in the algebra I assessment reliability because it does not contain CR items. No CR items were identified as low-performing or in need of consideration for exclusion.



**Table 7** Range of Item Difficulty

Assessment	N	Minimum % correct	Maximum % correct	N < 25% correct	N > 75% correct
Mathematics 4–5					
Multiple-choice items	14	11.6	73.3	4	0
Table items	24	27.7	96.0	0	8
Mathematics 6–8					
Multiple-choice items	14	9.4	67.6	3	0
Table items	32	35.4	98.1	0	16
Algebra I					
Multiple-choice items	19	14.0	76.9	3	2
Table items	18	30.1	100.0	0	11
ELA 4–6					
Multiple-choice items	17	26.9	91.9	0	4
Table items	34	33.3	96.0	0	17
ELA 7–9					
Multiple-choice items	16	20.3	81.6	1	2
Table items	32	10.5	99.3	2	18

Note. ELA = English language arts.

**Table 8** Comparison of Percent Correct on Common Subsets of Content Knowledge for Teaching (CKT) Items

Assessment forms	Items shared across forms	Percentage of items with average scores higher on higher-grade form
Mathematics 4–5 and Mathematics 6–8	9	100
Mathematics 6–8 and Algebra I	12	92
ELA 4–6 and ELA 7–9	19	79

Note. ELA = English language arts.

The results from the item exclusion analysis and inclusion of constructed response items are summarized in Table 3. The resulting assessment reliabilities range from 0.69 to 0.83.

### Comparing Item Difficulty Across Assessments

As the item level data suggest (Table 7), mathematics assessments were, in general, more difficult than the ELA assessments. No item on the ELA 4–6 assessment was answered correctly by less than 25% of respondents, and more than 40% of items on the ELA assessments were answered correctly by 75% or more of respondents. The mathematics 4–5 assessment contained the highest percentage of items (29%) that were answered correctly by less than 25% of respondents and the lowest percentage (21%) of items for which more than 75% of respondents answered correctly. However, the algebra assessment had proportions similar to the ELA assessments regarding items that were difficult or easy for the majority of participants.

### Comparing Item Difficulty Across Grade Levels

Teachers at different grade levels will have varying experience in teaching particular concepts. It is also true that teachers in middle school tend to have been more likely than elementary teachers to major in the content area that they are teaching (Gitomer, 2007). Thus, we explored whether consistent differences existed in performance between these two groups on the same CKT items shared across tests taken by lower-grade and higher-grade teachers (e.g., mathematics 4–5 vs. mathematics 6–8). We found a consistent pattern where a greater percentage of teachers at the higher-grade levels answered common items correctly compared with teachers at the lower-grade levels. This discrepancy in group performance is greater on common mathematics items than on ELA items (Table 8).

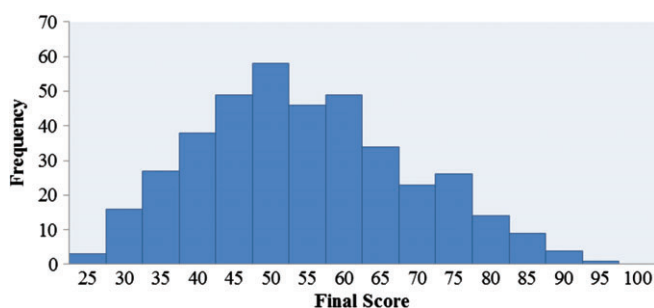
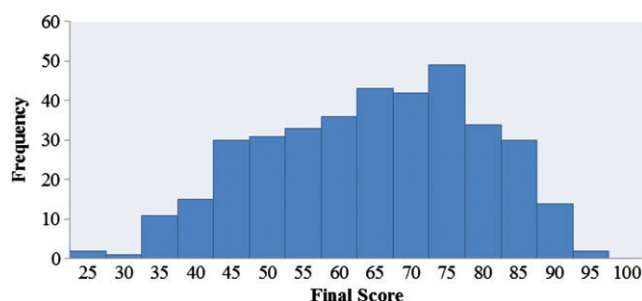
### Examining Score Distributions

Overall, mathematics assessments were more difficult than the ELA assessments, as indicated by the mean percent correct scores reported in Table 9.

**Table 9** Descriptive Statistics of Content Knowledge for Teaching (CKT) Assessment Scale Scores

Assessment	<i>n</i>	Minimum	Maximum	<i>M</i>	<i>SD</i>
Mathematics 4–5	397	20.5	93.2	52.2	14.4
Mathematics 6–8	373	22.6	94.2	62.1	14.9
Algebra I	143	22.9	97.1	61.5	14.3
ELA 4–6	555	30.8	89.4	66.4	11.7
ELA 7–9	418	31.4	88.7	64.7	10.5

Note. ELA = English language arts.

**Figure 3** Mathematics 4–5 score distribution (% correct).**Figure 4** Mathematics 6–8 score distribution (% correct).

To further examine the distribution of these scores across the assessment samples, we present score distributions on the percent correct scale scores in Figures 3–7. The distribution of scores is near normal for all assessments with a very broad range of scores. The mathematics 4–5 assessment distribution is positively skewed with a large number of low scores, and the ELA distributions are negatively skewed with a large number of high scores. The other mathematics assessments do not appear to be significantly skewed.

### Relationship Between Performances on Both Elementary Level Assessments

We designed the administration of the CKT assessments so that we could make comparisons across assessments. We analyzed the performance of teachers who completed both assessments at the elementary level in ELA and mathematics. After removal of unreliable scores, there remained a total of 271 teachers who had valid scores for both elementary assessments. We found a significant correlation between performances on both assessments ( $r = 0.49$ ,  $p < .01$ ). This relationship is represented graphically in Figure 8. It is also noteworthy that the ELA and mathematics assessments are clearly measuring different domains of knowledge, since the lack of correlation cannot be explained simply by uncorrelated measurement error (the correlation disattenuated for measurement error = 0.68).

Table 10 summarizes the performance on the mathematics and ELA CKT assessments by quartile. While quartile rank is similar for many teachers across assessments, there is a substantial number of teachers who are relatively strong on one test and relatively weak on another. For example, of the 85 teachers in the lowest math quartile, 17 (20%) have ELA scores in the upper two quartiles.

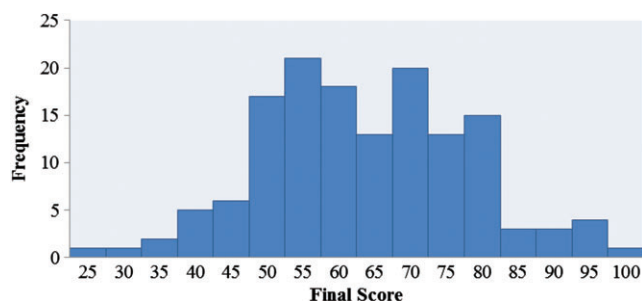


Figure 5 Algebra I score distribution (% correct).

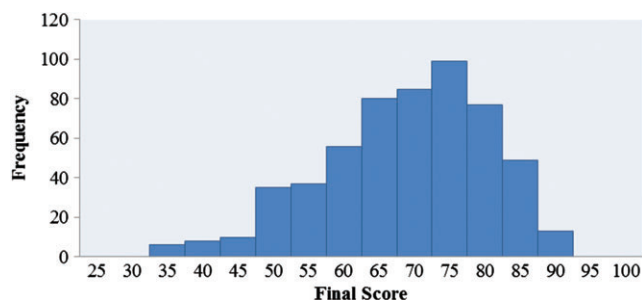


Figure 6 English language arts (ELA) 4–6 score distribution (% correct).

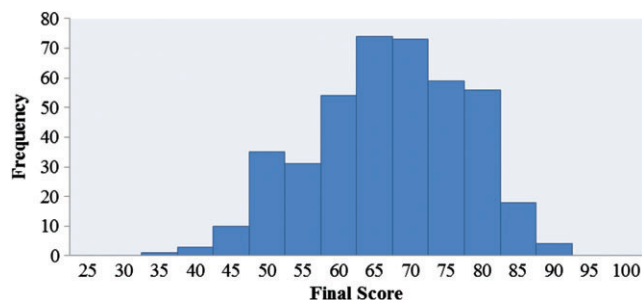


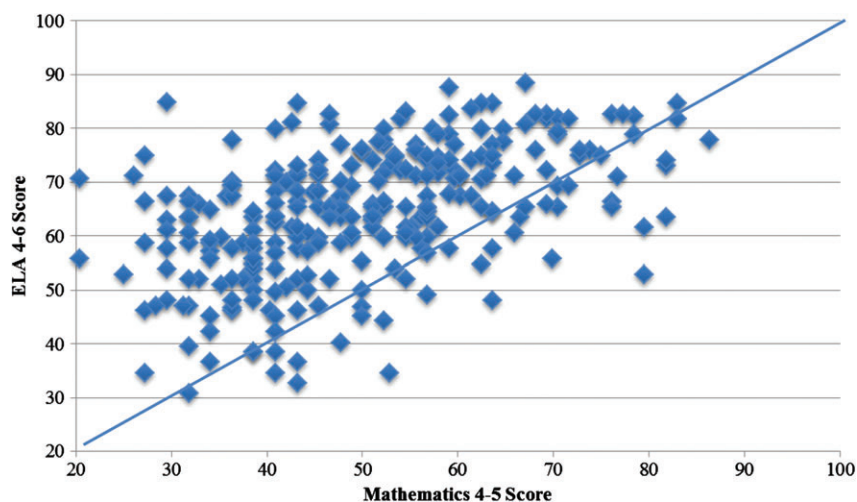
Figure 7 English language arts (ELA) 7–9 score distribution (% correct).

## Summary

The CKT development conducted as part of the MET study achieved a number of important goals. We highlight three accomplishments and findings: (a) the development of an assessment framework, (b) the generation of reliable assessments suitable for differentiating teachers across a range of CKT proficiency, and (c) practical considerations for how these assessments might be used to assess teacher quality.

Prior to the MET study, the assessment of CKT was carried out as part of research projects, primarily in mathematics, and largely without a well-articulated assessment design framework. One goal of the MET study was to propose a framework that could support identifying domains of CKT both within a given subject and across different subject areas. An important contribution is the CKT framework itself, along with the evidence supported by the development project that the framework can be used to support assessment development. The framework provides a starting point that can guide development efforts in other subjects and grade levels. The high-level categories provide a general organization that allows for making connections across different assessments. The lower-level, subject-specific categories provide examples that can guide the development of analogous or unique subject-specific tasks as needed. The CKT framework holds the promise of being generative for future assessment development.

The CKT framework also focuses attention on teaching. By definition, assessments of CKT are focused on the content knowledge that is needed to recognize, understand, or act on the content problems encountered in teaching. For this reason, CKT questions typically incorporate a teaching scenario or setting that test takers must consider in answering the



**Figure 8** Scores on elementary level assessment for teachers taking both the mathematics and English language arts (ELA) assessments.

**Table 10** Elementary Assessment Score Quartile Crosstabulation

Math scale score quartiles	ELA scale score quartiles				Total
	Q1	Q2	Q3	Q4	
Q1	51	17	14	3	85
Q2	18	23	16	8	65
Q3	8	16	25	16	65
Q4	5	12	14	25	56
Total	82	68	69	52	271

*Note.* ELA = English language arts.

content problem. Organizing the CKT assessment framework around tasks of teaching provides direct guidance for the types of teaching contexts that need to be represented in the assessment tasks. Our success in using the CKT framework to develop assessment questions across five tests in ELA and mathematics demonstrates the utility of a framework designed around tasks of teaching.

Another important contribution of this work is the actual set of assessments supported by the evidence that they can be used to generate reliable test scores that differentiate teachers across a wide range of the score distribution. The reliabilities for these five tests ranged from a low of 0.69 for Grades 7–9 ELA to a high of 0.83 for Grades 6–8 mathematics. While these reliabilities may seem low, it is important to realize that these assessments are comparable in their measurement quality to existing tests of teacher knowledge. The finding that the CKT tests for ELA were less reliable than for mathematics is also true for traditional content tests for teachers (ETS, 2010, p. 58), likely indicating domain difference rather than an inherent shortcoming of the CKT measures. Further, the CKT assessments are half the length of knowledge tests in current use. If the tests had been of equivalent lengths, the reliabilities would have been comparable to the more traditional knowledge tests, such as those from the *PRAXIS*<sup>®</sup> series. The CKT tests also provided adequate reliability across the wide range of ability. For all tests, a substantial number of teachers answered less than 50% of the items correctly and a substantial number answered more than 75% of the items correctly. This finding suggests that these tests would provide information on teachers' CKT across a large range of the knowledge continuum.

Finally, the results from the study also provide valuable information on how these assessments function in real school settings. The MET study involved teachers from six districts, a wide range of schools, and many different backgrounds. In a testing situation with no stakes attached, where teacher scores remained anonymous, and in which teachers were allowed to self-administer the assessment, evidence showed that the majority of teachers took these assessments seriously. Most teachers provided a good faith effort, spending the time needed to answer all assessment questions. This effort was true for teachers who received high scores and those who received low scores. This finding is encouraging, as it suggests that these assessments can be used to gather reliable information under conditions that do not include strong incentives for participation.

At the same time, a substantial number of teachers did not complete assessments as directed. On the elementary tests, nearly 15% of teachers appear to have completed the assessment with another teacher. This was the case even though teachers themselves were not being evaluated. It is unlikely that assessments such as these could be self-administered under conditions where teachers were being evaluated or other stakes were attached to their performance. In such cases, the number of collaborating teachers would likely increase along with the incentive to achieve a high score. Testing in situations where scores were public or used for consequential decisions would need to take place in a secure and proctored testing environment.

The results of this development effort are an assessment design framework and a set of assessments that reliably measures the types of content knowledge used in teaching practice. These assessments can be used to examine how measures of CKT are related to other measures of teaching effectiveness. They can be administered and studied in a variety of professional contexts. Future development of these assessments, or the creation of additional ones, can improve how we measure teaching quality and contribute to student learning.

### Acknowledgments

The authors would like to acknowledge the help from many people in the development of the content knowledge for teaching (CKT) assessments. Mark Thames led the initial work developing questions for the mathematics assessments. Barbara Weren was instrumental in all stages of the development, analysis, revisions, and administration of all assessments. Many individuals contributed to writing assessment questions, including Benjamin Baehr, Hyman Bass, Deanna Birdyshaw, Allison Brettschneider, Lindsay Brown, Leah Bullock, Matthew Burgess, Richard Chilcoat, Samantha Caughlan, Susan Dasch, Jenny DeMonte, Arne Jakobsen, Amy Johnson, Nadia Kalman, Minsung Kwon, Sarah Porter, Sarah Scott, and Elida Wylie. We also want to thank the following individuals who assisted with analysis, editing, study recruitment, and study administration: Michaela Arzt, Courtney Bell, Juana Betancourt, John Blackmore, Jason Bonthron, Andrew Croft, Michael Ecker, Jim Fife, Rebecca Gleeson, James Halliday, Barbara Hames, Erik Jacobson, Jyoti Kamal, Steve Meshanko, Anita O'Brien, Rob Rarich, Frank Rijmen, Melanie Schine, Jana Sukkarieh, Barbara Suomi, Amy Swauger, Sailesh Vezzu, and Mike Wagner.

### Notes

- 1 The distinction between organizing assessment by tasks of teaching rather than types of CKT is rather subtle. For a full discussion of this shift of perspective and what this particular approach offers for assessing content knowledge, see Phelps, Howell, and Kirui (2013).

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## Appendix A

## Mathematics 4–5 Assessment

Table A1 Item/Question Sequence Number for Mathematics 4–5 Assessment

Item/question sequence number	Answer key	Item/question sequence number	Answer key
1	A	10d	Provides
2	C	10e	Provides
3a	Not problematic	11	B
3b	Problematic	12	B
3c	Not problematic	13	B
3d	Not problematic	14a	Provides
3e	Problematic	14b	Does not provide
4	C	14c	Provides
5	A	14d	Provides
6	A	14e	Does not provide
7a	Provides	15	B
7b	Does not provide	16	B
7c	Provides	17	C
7d	Provides	18	C
7e	Provides	19a	Can
8	A	19b	Can
9	B	19c	Cannot
10a	Does not provide	19d	Can
10b	Provides	20	Open-ended response
10c	Does not provide	21	Open-ended response

Task

Question Number

MET1

1 of 21

Assessment Tools

Back

Review

Next

Hannah's Work

$$\begin{array}{r}
 9 \overline{)8,349} \\
 \underline{-8,100} \quad 900 \text{ 9s} \\
 249 \\
 \underline{-180} \quad 20 \text{ 9s} \\
 69 \\
 \underline{-63} \quad 7 \text{ 9s} \\
 6 \quad 927 \text{ 9s} \\
 \text{6 left over} \\
 \text{So } 8,349 \div 9 = 927 \text{ R}6
 \end{array}$$

Alan's Work

$$\begin{array}{r}
 2 \overline{)8,349} \\
 \underline{5} \\
 10 \\
 \underline{10} \quad 927 \\
 300 \\
 \underline{300} \\
 300 \\
 \underline{300} \\
 300 \\
 \underline{300} \\
 2,700 \\
 \underline{-2,700} \\
 2,949 \\
 \underline{-2,700} \\
 249 \\
 \underline{-90} \\
 159 \\
 \underline{-90} \\
 69 \\
 \underline{-45} \\
 24 \\
 \underline{-18} \\
 6 \\
 \text{So, you get 927} \\
 \text{with remainder 6.}
 \end{array}$$

Felipe's Work

$$\begin{array}{r}
 927 \\
 9 \overline{)8,349} \\
 \underline{-81} \\
 24 \\
 \underline{-18} \\
 69 \\
 \underline{-63} \\
 6
 \end{array}$$

After Mr. Ikler's class explored different strategies for dividing large numbers by single-digit divisors, they worked on the following problem.

Divide 8,349 by 9.

Students produced a variety of written strategies, some that are shown to the left.

Which of these students used a strategy that could be applied to correctly divide any two whole numbers?

☐ Hannah, Alan, and Felipe  
☐ Hannah and Alan only  
☐ Alan and Felipe only  
☐ Felipe only

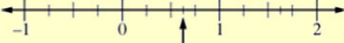
The answer is  $927 \frac{6}{9}$ .

Figure A1 Question #1.

Task	Question Number		Assessment Tools
MET1	2 of 21		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Castle gave her students a task that included the following question.

What value should be written where the arrow is pointing?



When correcting their papers, she noticed that many students gave an incorrect answer of  $\frac{3}{5}$  instead of the correct answer of  $\frac{5}{8}$ . Of the following, which is the most likely reason for this error?

- ☐ The students did not correctly identify the interval from 0 to 1 as the whole.
- ☐ The students counted the number of tick marks instead of counting the number of spaces between the tick marks.
- ☐ The students counted the numbers of intervals without regard to the length of the intervals.
- ☐ The students incorrectly identified the relevant benchmarks for this problem.

Figure A2 Question #2.

Task	Question Number		Assessment Tools
MET1	3 of 21		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Bowman found a supplemental worksheet on a Web site for teachers. The worksheet asked students to fill in one or more blanks in a mathematical statement with numbers that would make the statement true. However, she was concerned that some of the exercises on the worksheet were mathematically problematic because the blanks could not be filled in with numbers that would make the mathematical statement true. For each of the following, indicate whether or not the exercise is mathematically problematic.

	Mathematically Problematic	Not Mathematically Problematic
$8 + 15 = \underline{\quad} + 9$		
$14 + 5 = 19 + 5 = 24 + 5 = \underline{\quad}$		
$10 - 7 = 3 + \underline{\quad}$		
$29 - \underline{\quad} = 22 + 6 = 28$		
$6 - 2 = \underline{\quad} + 7 = \underline{\quad} + 5 = 16$		

Figure A3 Question #3.



Task	Question Number	Assessment Tools
MET1	4 of 21	<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Hupman is teaching an introductory lesson on exponents. She wants to give her students a quick problem at the end of class to check their proficiency in evaluating simple exponential expressions. Of the following expressions, which would be least useful in assessing student proficiency in evaluating simple exponential expressions?

- ☐  $3^3$
- ☐  $2^3$
- ☐  $2^2$
- ☐ All of these are equally useful in assessing student proficiency in evaluating simple exponential expressions.

Figure A4 Question #4.

Task	Question Number	Assessment Tools
MET1	5 of 21	<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Dixon is examining the school district's curriculum for teaching a concept of fraction. She wants to select some good introductory problems. Which of these problems involves the most elementary use of fractions?

- ☐ Jen has 8 cookies and 6 of them are chocolate chip cookies. What fraction of her cookies are chocolate chip cookies?
- ☐ Jen has 8 cookies and  $\frac{3}{4}$  of the cookies are chocolate chip cookies. How many of her cookies are chocolate chip cookies?
- ☐ Jen has 8 chocolate chip cookies and 6 oatmeal raisin cookies. What fraction of her cookies are chocolate chip cookies?
- ☐ All of the problems involve an equally elementary use of fractions.

Figure A5 Question #5.

Task	Question Number		Assessment Tools
MET1	6 of 21		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

In a unit on division, Ms. Jimenez gave her students the following problem.

Carlos wants to cover the bottom edge of his window with a row of tiles that are each 5 inches long. If the bottom of the window is 42 inches long, how many tiles will he need to buy?

When the students finished, she wanted to give them another problem that was mathematically similar to the first. Of the following, which is most similar to the original problem, both in terms of the meaning of division and the nature of the remainder?

- ☐ Tim needs to pack 42 binders into boxes in order to ship them. If each box can hold 5 binders, what would be the fewest number of boxes needed to ship all of the binders?
- ☐ Gabriela has 42 stickers and wants to divide them up equally among 5 of her friends. How many stickers should each friend get?
- ☐ Robert wants to wrap gifts, and he has 42 feet of ribbon on a spool. If each gift requires 5 feet of ribbon, how many gifts can be wrapped using the ribbon on the spool?
- ☐ Teresa has 42 inches of string. If she cuts the string into 5 equal pieces, how long will each piece be?

Figure A6 Question #6.

Task	Question Number		Assessment Tools
MET1	7 of 21		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

In a unit on proportional reasoning, Ms. Richmond's class was discussing the following problem.

If 4 cups of cocoa and 2 cups of sugar yield 16 brownies, how many cups of cocoa and how many cups of sugar are needed to make 24 brownies?

Ms. Richmond's students used different strategies to solve the problem. For each strategy, indicate whether or not it provides evidence of mathematically valid student thinking.

	Provides Evidence of Mathematically Valid Student Thinking	Does Not Provide Evidence of Mathematically Valid Student Thinking
48 brownies need 12 cups of cocoa and 6 cups of sugar. To make 24 brownies, I need 6 cups of cocoa and 3 cups of sugar.	<input type="checkbox"/>	<input type="checkbox"/>
4 and 2 both go into 16. 4 plus 2 is 6, half of 6 is 3, and 6 and 3 both go into 24, so you need 6 cups of cocoa and 3 cups of sugar to make 24 brownies.	<input type="checkbox"/>	<input type="checkbox"/>
1 brownie needs $\frac{1}{4}$ cup of cocoa and $\frac{1}{8}$ cup of sugar. To make 24 brownies, I need to multiply by 24 for cocoa and sugar. Thus, I need 6 cups of cocoa and 3 cups of sugar.	<input type="checkbox"/>	<input type="checkbox"/>
6 cups of cocoa and sugar makes 16 brownies, so 24 brownies need 9 cups of cocoa and sugar. Since the ratio of cocoa to sugar is 2:1, I need 6 cups of cocoa and 3 cups of sugar.	<input type="checkbox"/>	<input type="checkbox"/>
Since 1 cup of sugar is needed to make 8 brownies, I need 3 cups of sugar to make 24 brownies. The amount of cocoa is two times the amount of sugar in the recipe, so I need 6 cups of cocoa.	<input type="checkbox"/>	<input type="checkbox"/>

Figure A7 Question #7.

Task	Question Number		Assessment Tools
MET1	8 of 21		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Malloy's textbook suggests using white chips (for positive integers) and red chips (for negative integers) to teach integer subtraction. To solve the problem  $2 - (-3)$ , the book describes the process below.

1. Start with two white chips.
2. Add three pairs of red and white chips.
3. Then take away three red chips for the  $-3$ .
4. There are 5 white chips left, so the answer is 5.

Ms. Malloy is concerned that her students will see that adding red and white chips sets up the subtraction, but not understand why the additional chips can be added. Of the following, which provides a mathematical explanation for why the three pairs of red and white chips can be added?

- ☐ By adding both a red chip and a white chip to an expression, you are really adding a zero, which does not change the value of the original amount.
- ☐ The 2 is two white chips and the  $-3$  is three red chips. When you add the three red chips, you also add three white chips. That way, when you take away the red chips, the total number of white chips will be 5, which is the correct answer.
- ☐ The three red chips are for the  $-3$  and the three white chips are what the red chips become because the negative of a negative is positive.
- ☐ Red chips represent negative integers, but when you subtract negative three, you get positive three, so you need to add three white chips as well as three red chips.

Figure A8 Question #8.

Task	Question Number		Assessment Tools
MET1	9 of 21		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Mr. Chamberlain is concerned that his students' use of the calculator has led them to view the equal sign as a signal to carry out an operation rather than as a symbol indicating equality. Of the following missing-number problems, which would best assess whether students understand the mathematically correct meaning of the equal sign?

- ☐  $\_\_ + \_\_ = 18$
- ☐  $7 + 5 = \_\_ + 6$
- ☐  $\_\_ = 17 + 9 + 5$
- ☐  $23 + 4 = \_\_ = 4 + 23$

Figure A9 Question #9.

Task	Question Number	Assessment Tools	
MET1	10 of 21	<div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;">Back</div> <div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;">Review</div> <div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;">Next</div>	

Mr. Lee asked his students to compare  $\frac{7}{8}$  and  $\frac{6}{9}$ . All of his students correctly answered that  $\frac{7}{8}$  is greater than  $\frac{6}{9}$ , but they offered a variety of responses when asked to explain their reasoning. Of the following, which student responses provide mathematically valid explanations for why  $\frac{7}{8}$  is greater than  $\frac{6}{9}$ ?

For each student response, indicate whether or not it provides a mathematically valid explanation.

	Provides a Mathematically Valid Explanation	Does Not Provide a Mathematically Valid Explanation
When you compare them, $\frac{7}{8}$ is greater than $\frac{6}{9}$ because 7 is greater than 6.		
You can see that $\frac{7}{8}$ is greater than $\frac{6}{9}$ because ninths are smaller than eighths, which means that $\frac{6}{9}$ is less than $\frac{6}{8}$ which is less than $\frac{7}{8}$ .		
You just need to look at how many pieces are missing. $\frac{7}{8}$ is greater than $\frac{6}{9}$ because $\frac{7}{8}$ is only missing one piece from the whole, but $\frac{6}{9}$ is missing three pieces from the whole.		
I think $\frac{7}{8}$ is greater than $\frac{6}{9}$ because $\frac{7}{8}$ has more pieces than $\frac{6}{9}$ and those pieces are larger.		
$\frac{7}{8}$ is greater than $\frac{6}{9}$ because $\frac{6}{9}$ is equal to $\frac{2}{3}$ , and because $\frac{1}{3}$ is greater than $\frac{1}{8}$ , $\frac{2}{3}$ is farther away from 1 than $\frac{7}{8}$ is.		

Figure A10 Question #10.

Task	Question Number	Assessment Tools	
MET1	11 of 21	<div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;">Back</div> <div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;">Review</div> <div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;">Next</div>	

To assess her students' prior knowledge about evaluating arithmetic expressions, Ms. Santiago assigned a worksheet of problems. She noticed that Alexis answered the first two incorrectly and the next two correctly.

1)  $7 \times 2 - 6 + 3 = 5$

2)  $9 - 5 + (16 \div 8) = 2$

3)  $9 + 24 \div 3 - 1 = 16$

4)  $17 - (3 + 7 \times 2) = 0$

Which of the remaining problems is Alexis likely to answer incorrectly?

☐  $8 + 7 - 12 \div 3$   
☐  $13 - 3 \times 2 + 5$   
☐  $(27 \div 3 - 4) + 8$   
☐  $(16 - 12) \times 5 + 10$

Figure A11 Question #11.

Task	Question Number		Assessment Tools
MET1	12 of 21	<p>Ms. Jones's class has been studying area and perimeter. Some of the students in her class are learning English; they speak Spanish at home and when they work in small groups. One day they work on the following problem.</p> <ul style="list-style-type: none"> <li>Look for all of the rectangles with area 36 (where length and width are integers). Write down the dimensions.</li> <li>Calculate the perimeter for each rectangle.</li> <li>Describe a pattern relating the perimeter and the dimensions.</li> </ul> <p>Of the following student responses, which most accurately describes the relationship between the dimensions of a rectangle and its perimeter?</p> <ul style="list-style-type: none"> <li><input type="radio"/> A rectangle has ... two ... short sides, and two long sides. The perimeter of the ... long ... skinny rectangle is big. It's about twice longer than the square.</li> <li><input type="radio"/> The longer the ah ... the longer [traces the shape of a long rectangle with her hands several times] the ah ... the longer the, thing, you know the more the perimeter, the higher the perimeter is.</li> <li><input type="radio"/> The rectangle ... the one with 4 width and 9 length ... its perimeter 26, and its area is bigger, it's 36. The rectangle 3 width and 12 length ... it's perimeter 30. It's 4 more ... it goes up by 4 ... higher.</li> <li><input type="radio"/> Each of these student responses describes the relationship with equal accuracy.</li> </ul>	<div style="border: 1px solid black; padding: 2px; display: inline-block;"> <div style="display: flex; justify-content: space-between; width: 100%;"> <span>Back</span> <span>Review</span> <span>Next</span> </div> </div>

Figure A12 Question #12.

Task	Question Number		Assessment Tools
MET1	13 of 21	<p>During an introductory unit on integers, Ms. Guerra asked her students to suggest a sentence illustrating what the word "opposite" meant to them. She wanted to use one or two of their suggestions as images to build on as they learned about integers. Of the following student suggestions, which gives the <u>LEAST</u> accurate illustration of the mathematical meaning of the word "opposite" as it is used in the context of integers?</p> <ul style="list-style-type: none"> <li><input type="radio"/> Profit is the opposite of loss.</li> <li><input type="radio"/> Entrance is the opposite of exit.</li> <li><input type="radio"/> Clockwise is the opposite of counterclockwise.</li> <li><input type="radio"/> All of the student suggestions give equally accurate illustrations of the mathematical meaning of the word "opposite."</li> </ul>	<div style="border: 1px solid black; padding: 2px; display: inline-block;"> <div style="display: flex; justify-content: space-between; width: 100%;"> <span>Back</span> <span>Review</span> <span>Next</span> </div> </div>

Figure A13 Question #13.

Task	Question Number			Assessment Tools
MET1	14 of 21			<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Mr. Hinton gives his students the following division problem to solve.

$1056 \div 12$

As he moves around the room, he notices his students using several different strategies. For each of the following student solutions, indicate whether or not the work provides evidence that the student is reasoning correctly about this problem.

	Provides Evidence of Correct Student Reasoning	Does Not Provide Evidence of Correct Student Reasoning
$\begin{array}{r} 120 \ 10 \\ 720 \ 50 \\ 960 \ 20 \\ 1020 \ 5 \\ 1056 \ 3 \\ \hline 88 \end{array}$		
$\begin{array}{r} 1000 \div 10 \ 100 \\ 100 - 12 \ 88 \end{array}$		
$\begin{array}{r} 1200 \ 100 \\ 120 \ 10 \\ \hline 1080 \ 90 \\ 24 \ 2 \\ \hline 1056 \ 88 \end{array}$		
$\begin{array}{r} 960 \div 12 \ 80 \\ 96 \div 12 \ 8 \\ \hline 1056 \div 12 \ 88 \end{array}$		
$\begin{array}{l} 1000 \div 10 = 100 \\ 56 \div 2 = 28 \\ \rightarrow 100 - 20 + 8 = 88 \end{array}$		

Figure A14 Question #14.

Task	Question Number			Assessment Tools
MET1	15 of 21			<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

At the start of a lesson on finding the side length of a square given its area, Ms. Sabatine gave her students a problem to assess their prior knowledge. Several students incorrectly answered that the side length of a square with area 36 square units is 9 units. At the end of the lesson, Ms. Sabatine wanted to give a similar problem to assess what her students had learned. Of the following areas to use in this problem, which would be least useful for assessing student learning in this situation?

- ☐ 9 square units
- ☐ 16 square units
- ☐ 64 square units
- ☐ 100 square units

Figure A15 Question #15.

Task	Question Number		Assessment Tools
MET1	16 of 21		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

During a unit on decimals, Shanti claimed she knew an easy way to divide by ten.

To divide by ten, you just move the decimal point one place to the left. For example:  $427.3 \div 10 = 42.73$ .

Ms. Burns asked her students to explain why Shanti's rule works. Which of the following student responses provides the best evidence that the student understands why Shanti's rule works?

- ☐ Shanti's rule works because if you multiply the answer by 10 it moves the decimal point one place to the right and you get the starting number.
- ☐ Shanti's rule works because dividing a number by 10 divides the value for each digit by 10, which is the same as moving all of the digits over one place to the right.
- ☐ Shanti's rule works because when you divide you have to add the number of decimal places in the dividend and the number of ending zeros in the divisor—so with 10 it's one zero, which is one additional decimal place or just moving the decimal point one to the left.
- ☐ These all provide comparable evidence that the students understand why Shanti's rule works.

Figure A16 Question #16.

Task	Question Number		Assessment Tools
MET1	17 of 21		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

A lesson in Ms. Taylor's textbook states the associative and commutative properties of addition. To motivate the students to learn the properties, she tells her students that the properties can often be used to simplify the evaluation of expressions.

She wants to give her students an example that will focus their attention on how these properties can be useful in evaluating expressions. Of the following expressions, which would best serve her purpose?

- ☐  $(455 + 456) + (457 + 458)$
- ☐  $(647 + 373) + (227 + 456)$
- ☐  $(551 + 775) + (49 + 225)$
- ☐ Each of these expressions would serve her purpose equally well.

Figure A17 Question #17.



Task	Question Number	Assessment Tools
MET1	18 of 21	<div style="display: flex; justify-content: flex-end; gap: 5px;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Schrader is teaching her class how to translate verbal expressions into algebraic expressions. Some of the students in her class speak Spanish at home and when they work in small groups. Of the following verbal expressions that translate into  $x - 5$ , which would be the most difficult for her Spanish-speaking students to translate correctly?

- ☐ A number minus 5
- ☐ The difference of a number and 5
- ☐ 5 less than a number
- ☐ Each of these expressions would present a similar level of difficulty for Spanish-speaking students to translate.

Figure A18 Question #18.

Task	Question Number	Assessment Tools
MET1	19 of 21	<div style="display: flex; justify-content: flex-end; gap: 5px;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Mr. Xavier is teaching his students to solve word problems. Students are learning to draw pictures to help them reason about the situations and about the computations used to solve the problems. Mr. Xavier wants to give his students problems where each problem has a figure and a computation and have his students discuss in small groups whether the figure can be used to model the given computation. For each of the following, indicate whether the figure **can** or **cannot** be used to model the given computation.

Figure	Computation	Can Be Used	Cannot Be Used
	$12 \div 3$		
	$1\frac{1}{2} \div \frac{1}{6}$		
	$48 \div \frac{5}{8}$		
	$1\frac{1}{2} \div \frac{1}{3}$		

Figure A19 Question #19.

Task	Question Number		Assessment Tools
MET1	20 of 21		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Franco was assessing students' work on comparing fractions. She assigned the following problem.

Put the following fractions in increasing order and explain your reasoning.  $\frac{4}{7}, \frac{5}{8}, \frac{2}{5}$

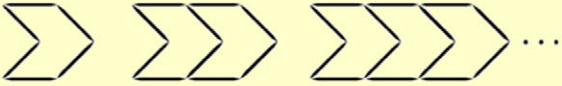
She noticed that Zachary got a correct answer with incorrect reasoning.  
 He explained that  $\frac{2}{5} < \frac{4}{7} < \frac{5}{8}$  because  $2 < 4 < 5$  and  $5 < 7 < 8$ .

To help Zachary understand that his reasoning is incorrect, Ms. Franco wants to give a similar problem using 3 different fractions. She wants to include fractions with 3 different numerators and 3 different denominators that, using Zachary's reasoning, would lead to ordering the fractions incorrectly, from greatest to least instead of least to greatest. List 3 such fractions in the boxes below in any order.


Figure A20 Question #20.

Task	Question Number		Assessment Tools
MET1	21 of 21		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Mr. Colwell gave his students the following problem.



The first figure is made up of 6 toothpicks. The second is made up of 10 toothpicks. Use the variable  $n$  to write an expression to show the number of toothpicks in any figure.

Ian's expression was  $4n + 2$ .  
 Explain what the variable  $n$  in Ian's response most likely represents.

Samantha's expression was  $n + 4$ .  
 Explain what the variable  $n$  in Samantha's response most likely represents.

Figure A21 Question #21.

## Appendix B

## English Language Arts (ELA) 4–6 Assessment

Table B1 Item/Question Sequence Number for English Language Arts (ELA) 4–6 Assessment

Item/question sequence number	Answer key	Item/question sequence number	Answer key
1	B	12d	Would not
2	A	12e	Would
3	C	13	C
4	B	14	D
5	A	15	B
6a	Yes	16a	Effective
6b	Likely	16b	Effective
6c	Likely	16c	Effective
6d	Not likely	16d	Not effective
6e	Likely	16e	Not effective
6f	Not likely	17	B
7	C	18	Open-ended response
8a	Will	19a	Will help
8b	Will not	19b	Will help
8c	Will	19c	Will not help
8d	Will not	19d	Will not help
9	Accurate	20	B
10	D	21	C
11a	Accurate	22	Open-ended response
11b	Accurate	23a	Does not describe
11c	Not accurate	23b	Does not describe
11d	Accurate	23c	Describes
11e	Not accurate	23d	Describes
11f	Not accurate	24	A
12a	Would not	25	D
12b	Would	26	B
12c	Would		

Task	Question Number	Assessment Tools
MET1	1 of 26	Back Review Next

Mr. Kowalski and his sixth-grade students are in literature circles discussing *A Year Down Yonder*, a historical novel about life in a small town in Depression-era Illinois. He notices that his students' conversations are stilted and recitation-like.

To best help his students have more productive conversations in literature circles, Mr. Kowalski should encourage them to

- ☐ nominate a discussion leader who would ensure everyone participates
- ☐ build on each others' ideas by asking follow-up questions
- ☐ focus on assigned roles for the literature circles
- ☐ participate in unstructured "talk time" so that they become comfortable speaking to their peers

Figure B1 Question #1.

Task	Question Number		Assessment Tools
MET1	2 of 26	<p>Ms. Reinhardt has her sixth- and seventh-grade students read the following poem by Emily Dickinson.</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <p><b>I'm nobody! Who are you?</b></p> <p>I'm nobody! Who are you?          Are you nobody, too?          Then there's a pair of us—don't tell!          They'd banish us, you know.</p> <p>How dreary to be somebody!          How public, like a frog          To tell your name the livelong day          To an admiring bog!</p> </div> <p>To make the poem's meaning more accessible, Ms. Reinhardt wants to relate the poem to a topic of interest to her students. She decides to focus on teen celebrities, a current hot topic in her classroom.</p>	<div style="border: 1px solid black; padding: 5px;"> <p>Which of the following is the best option to help students use their knowledge about teen celebrities to understand this poem?</p> <p>Ms. Reinhardt should have students think about the way teen celebrities are</p> <ul style="list-style-type: none"> <li><input type="radio"/> often on display, with the media focusing on their every move</li> <li><input type="radio"/> admired by the public regardless of their positive or negative contributions</li> <li><input type="radio"/> celebrated in the media for a short time but then hardly thought about</li> <li><input type="radio"/> sometimes immature and therefore not able to handle the pressures of fame</li> </ul> </div>

Figure B2 Question #2. *I'm Nobody* by Emily Dickinson. Reprinted by permission of Dover Publications.

Task	Question Number		Assessment Tools
MET1	3 of 26	<p>While students are reading independently, Ms. Kumar begins talking with Doug, a struggling fourth-grade reader, about the following two sentences he just read from a book about a girl and her dog.</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <p>I have a dog named Glover. That mischievous mutt is always getting into trouble.</p> </div> <p>Doug comments that his dog is bad too, and that he is excited to read more about the two dogs in the story. Ms. Kumar is trying to figure out why Doug is misreading the text.</p> <p>Which of the following is the best explanation for Doug's confusion?</p> <ul style="list-style-type: none"> <li><input type="radio"/> Doug has not comprehended the word "mischievous."</li> <li><input type="radio"/> Doug has made a connection to his dog, which has interfered with his comprehension.</li> <li><input type="radio"/> Doug does not understand that "Glover" and "mischievous mutt" refer to the same dog.</li> <li><input type="radio"/> Doug is confused by the complex sentence structures used in the text.</li> </ul>	<div style="border: 1px solid black; padding: 5px;"> <p>Back Review Next</p> </div>

Figure B3 Question #3.

Task	Question Number		Assessment Tools
MET1	4 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 4-5 are based on the following teaching scenario.*

Mr. Goldberg is teaching the science fiction novel *The Giver*, by Lois Lowry, to his sixth- and seventh-grade students. The people in the society presented in the novel have no knowledge of the world outside their community. Only one person in the society, known as "the Giver," holds knowledge of the rest of world. The main character in the novel, Jonas, is designated as the Giver at age 12. In the passage below, Jonas is startled by the look of a newborn baby's eyes. Mr. Goldberg reads the passage aloud to the class:

But he had been startled by the newchild's eyes. Mirrors were rare in the community; they weren't forbidden, but there was no real need of them, and Jonas had simply never bothered to look at himself very often even when he found himself in a location where a mirror existed. Now, seeing the newchild and its expression, he was reminded that the light eyes were not only a rarity but gave the one who had them a certain look—what was it? *Depth*, he decided; as if one were looking into the clear water of the river, down to the bottom, where things might lurk which hadn't been discovered yet. He felt self-conscious, realizing that he, too, had that look.

Mr. Goldberg stops reading when he realizes that some students seem confused.

Which of the following activities would best help Mr. Goldberg's students comprehend the passage?

- ☐ Visualize Jonas and the "newchild" looking at each other.
- ☐ Use evidence from the text to show how Jonas was feeling.
- ☐ Focus on Lowry's use of italics to emphasize certain words.
- ☐ Describe a time when you felt self-conscious like Jonas.

Figure B4 Question #4.

Task	Question Number		Assessment Tools
MET1	5 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 4-5 are based on the following teaching scenario.*

Mr. Goldberg is teaching the science fiction novel *The Giver*, by Lois Lowry, to his sixth- and seventh-grade students. The people in the society presented in the novel have no knowledge of the world outside their community. Only one person in the society, known as "the Giver," holds knowledge of the rest of world. The main character in the novel, Jonas, is designated as the Giver at age 12. In the passage below, Jonas is startled by the look of a newborn baby's eyes. Mr. Goldberg reads the passage aloud to the class:

But he had been startled by the newchild's eyes. Mirrors were rare in the community; they weren't forbidden, but there was no real need of them, and Jonas had simply never bothered to look at himself very often even when he found himself in a location where a mirror existed. Now, seeing the newchild and its expression, he was reminded that the light eyes were not only a rarity but gave the one who had them a certain look—what was it? *Depth*, he decided; as if one were looking into the clear water of the river, down to the bottom, where things might lurk which hadn't been discovered yet. He felt self-conscious, realizing that he, too, had that look.

Which of the following questions would best focus a class discussion on the theme of the passage?

- ☐ Why is Jonas startled when he looks into the "newchild's" eyes?
- ☐ Why is Jonas chosen to be the Giver?
- ☐ Why are there so few mirrors in the community?
- ☐ Why has Jonas never bothered to look at himself in a mirror?

Figure B5 Question #5.



Task	Question Number		Assessment Tools
MET1	6 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Mr. Wong and his fifth-grade students are studying the impact of humans on the environment. The students in Mr. Wong's class are avid readers, often discussing and writing about fiction that they read. Because his students are strong readers, Mr. Wong is surprised that many of them struggle to comprehend informational text. He gives his students the following passage to read in pairs and notes that they can read it fluently.

**Save the Soil!**

Soil seems to be everywhere—in fields, in backyards, clinging to the soles of our shoes. But the nation's fertile soil is vanishing at an alarming rate. For every bushel of corn produced in the U.S., about a bushel of soil disappears.

How? Rain can wash loose soil into streams and rivers. To replace an inch of washed-away topsoil, plants and other matter on the surface must break down for hundreds or even thousands of years. Soil disappears when houses or malls are built on land where crops could be planted. Pollution also ruins soil, making it unsafe for planting.

In the discussion that follows, many students seem to have trouble comprehending the information about soil.

For each text feature, indicate whether it will be likely to cause difficulty.

	Likely to cause difficulty	Not likely to cause difficulty
The connection between "fertile soil" and "bushel of corn produced"		
The synonymous use of "fertile soil" and "topsoil"		
Starting the second paragraph with the one-word sentence "How?"		
The lack of an explicit statement that humans play a role in erosion		
The use of subordinate clauses in a number of sentences		
The description in the first sentence of the text of where the soil might be found		

**Figure B6** Question #6. Adapted from *TIME for Kids Magazine*, 2/9/1999 © 1999 Time Inc. Used under license. TIME for Kids and Time Inc. are not affiliated with, and do not endorse, products or services of Educational Testing Service.

Task	Question Number		Assessment Tools
MET1	7 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Next question

*Questions 7-9 are based on a lesson using Jerry Spinelli's novel "Maniac Magee."*

Mr. Haddad is using Jerry Spinelli's novel *Maniac Magee* to teach his fifth-grade class about how authors develop their characters.

Which of the following questions would be the best choice to help his students focus on the essential features of character development?

- ☐ What kinds of symbolism does the author use to develop Maniac Magee's character?
- ☐ Can you describe where Maniac Magee lives?
- ☐ How does Maniac Magee respond to trouble?
- ☐ What do you think might have happened to Maniac Magee before this story began?

**Figure B7** Question #7.



<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
<b>MET1</b>	<b>8 of 26</b>		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

[Next question](#)

Questions 7-9 are based on a lesson using Jerry Spinelli's novel *"Maniac Magee."*

Mr. Haddad's class continues to focus on character development while reading *Maniac Magee*.

For each activity, indicate whether it would be effective in assessing students' understanding of how an author develops a character over the course of a novel.

	Activity will assess students' understanding	Activity will not assess students' understanding
Document changes to the main character using a time line.		
Draw a detailed picture of the character.		
Keep a journal of the novel's main events from the character's point of view.		
Have students share their own attitudes toward the character with the class.		

Figure B8 Question #8.

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
<b>MET1</b>	<b>9 of 26</b>		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Questions 7-9 are based on a lesson using Jerry Spinelli's novel *"Maniac Magee."*

Mr. Haddad asks his students to create a character map—a graphic organizer in which students record words and short phrases that capture the most important aspects of a character's actions and memorable sayings, as well as how other people react to the character. Rachel has chosen John McNab, a minor character who appears infrequently in *Maniac Magee*. Mr. Haddad notices that Rachel has been copying every word in the novel about John McNab into her notebook.

Which of the following is the best teaching approach to help Rachel record useful information about her character's development?

- ☐ Interview her about what John McNab says and does at important points in the story.
- ☐ Suggest that she use a character map of *Maniac Magee* as a model for her own work.
- ☐ Show her how to discriminate between important and unimportant information in text.
- ☐ Help her create a list of events that involve John McNab, and model how to write about the first event.

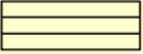
Figure B9 Question #9.

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
<b>MET1</b>	<b>10 of 26</b>		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

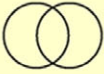
*Questions 10-11 are based on the following teaching scenario.*

Mr. Foreman is teaching his sixth- and seventh-grade students a unit that focuses on strategies for reading informational text. His textbook includes a resource section with ideas that can help students identify the organization of text passages. The following graphic organizers are included in the textbook's resource section:

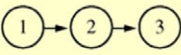
**Description or List**



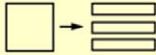
**Compare and Contrast**



**Sequence/Time order**



**Cause and Effect**



Mr. Foreman is considering how to use these examples to help his students understand how graphic organizers can support their comprehension of informational text.

To best help his students improve their reading of informational text, Mr. Foreman should have them use these graphic organizers to

- ☐ record and focus on the meaning of key vocabulary in the text
- ☐ comprehend that nonfiction texts are used to extract factual information
- ☐ identify concrete visual images to recall important details
- ☐ anticipate patterns of information by identifying commonly used structures

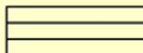
Figure B10 Question #10.

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
<b>MET1</b>	<b>11 of 26</b>		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

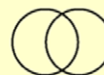
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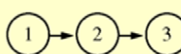
**Description or List**



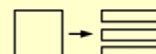
**Compare and Contrast**



**Sequence/Time order**



**Cause and Effect**



Mr. Foreman is considering how to use these examples to help his students understand how graphic organizers can support their comprehension of informational text.

Mr. Foreman is thinking about helpful information to share with his students about these graphic organizers.

For each claim, indicate whether it is an accurate statement.

	Accurate statement	Not an accurate statement
"Cause and Effect" cannot always be used because effects sometimes precede causes in an essay.		
Sometimes more than one graphic organizer can be used because some essays have multiple text structures.		
"Compare and Contrast" is the most useful graphic organizer for nonfiction.		
There are more nonfiction text structures than those presented by these graphic organizers.		
"Description or List" and "Sequence/Time Order" are always chronological.		
These graphic organizers can be used interchangeably.		

Figure B11 Question #11.

Task	Question Number		Assessment Tools
MET1	12 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Jackman has noticed that the English-language learners in her classroom are proficient in conversational English and skilled at reading fiction. However, these students have difficulty reading textbooks in science and social studies and generally do not comprehend those texts as well. Ms. Jackman's class is about to read the following text as part of a unit on endangered species.

Cougars, often called mountain lions, are solitary and secretive animals that are seldom seen in the wild. Cougars are known for their strength, agility, and ability to jump. Their powerful legs allow them to leap 30 feet from a standstill, or to jump 15 feet straight up a cliff wall. A cougar's strength and powerful jaws allow it to take down and drag prey larger than itself.

Cougars are the largest members of the cat family in North America. Adult males weigh between 140 and 180 pounds and measure 7-8 feet long from nose to tip of tail. They vary in color from reddish-brown to gray, with a black tip on their long tail.

Ms. Jackman reads the text in advance to anticipate the difficulties her English-language learners may encounter.

For each text feature, indicate whether it would represent a reading challenge for the English-language learners in Ms. Jackman's class.

	Would represent a reading challenge for English-language learners	Would not represent a reading challenge for English-language learners
Text not in chronological order		
Words like "wild" and "mountain"		
Words like "seldom" and "often"		
Sentences without the word "cougar," e.g., "Their powerful legs..." and "They vary in color..."		
References to units of measure in feet and pounds		

Figure B12 Question #12.

Task	Question Number		Assessment Tools
MET1	13 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Meyer is reading her fourth-grade students' papers to identify short passages that demonstrate good use of details in their writing.

Which of the following samples of student writing would be best to share with the class as a model?

- ☐ When my mother gets angry she yells a lot, and then her voice is loud like a siren.
- ☐ The big door was shaped like a rectangle, and it was made of fancy wood.
- ☐ She jumped so fast that her feet thumped the way a wild rabbit sounds hopping in a cage.
- ☐ I went downstairs for breakfast, sat down at the table in a comfortable chair, and ate a delicious muffin.

Figure B13 Question #13.

Task	Question Number		Assessment Tools
MET1	14 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 14-17 are based on the following teaching scenario.*

Ms. Garcia is working with an advanced sixth-grade reading group. She is using the following excerpt from *Their Eyes Were Watching God* by Zora Neale Hurston as a model for a writing assignment.

So Janie began to think of Death. Death, that strange being with the huge square toes who lived way in the West. The great one who lived in a straight house like a platform without sides to it, and without a roof. What need has Death for a cover, and what winds can blow against him? He stands in his high house that overlooks the world. Stands watchful and motionless all day with his sword drawn back, waiting for the messenger to bid him come. Been standing there before there was a where or a when or a then. She was liable to find a feather from his wings lying in her yard any day now.

Ms. Garcia has recently been working on a number of literary techniques with her students. Before beginning the lesson, Ms. Garcia looks over the paragraph.

Which of the following best describes the central literary technique used in this passage?

- ☐ Simile
- ☐ Alliteration
- ☐ Rhetorical question
- ☐ Personification

Figure B14 Question #14.

Task	Question Number		Assessment Tools
MET1	15 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 14-17 are based on the following teaching scenario.*

Ms. Garcia is working with an advanced sixth-grade reading group. She is using the following excerpt from *Their Eyes Were Watching God* by Zora Neale Hurston as a model for a writing assignment.

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During a discussion of the passage, Ms. Garcia asks her students, "What effect do you think Hurston is trying to achieve in the last sentence?"

Which of the following student responses provides the best explanation of the author's use of the feather in the passage?

- ☐ "It probably means the person is like a Greek god."
- ☐ "I think it is saying that the death guy is close by."
- ☐ "It probably is meant as a symbol of beauty to help Janie stop thinking about death."
- ☐ "I think she wants to get away or fly away from home like a bird."

Figure B15 Question #15.



Task	Question Number		Assessment Tools																		
MET1	16 of 26	<p><i>Questions 14-17 are based on the following teaching scenario.</i></p> <p>Ms. Garcia is working with an advanced sixth-grade reading group. She is using the following excerpt from <i>Their Eyes Were Watching God</i> by Zora Neale Hurston as a model for a writing assignment.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>So Janie began to think of Death. Death, that strange being with the huge square toes who lived way in the West. The great one who lived in a straight house like a platform without sides to it, and without a roof. What need has Death for a cover, and what winds can blow against him? He stands in his high house that overlooks the world. Stands watchful and motionless all day with his sword drawn back, waiting for the messenger to bid him come. Been standing there before there was a where or a when or a then. She was liable to find a feather from his wings lying in her yard any day now.</p> </div> <p>Ms. Garcia asks students to write a poem that mimics Hurston's use of abstract nouns in the passage.</p> <p>For each teaching method, indicate whether it would be effective in helping students generate ideas for a poem that mimics Hurston's use of abstract nouns.</p> <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <thead> <tr> <th></th> <th style="text-align: center;">Effective</th> <th style="text-align: center;">Not Effective</th> </tr> </thead> <tbody> <tr> <td>Help students brainstorm a list of abstract nouns other than "death" and write the students' suggestions on the board.</td> <td></td> <td></td> </tr> <tr> <td>Share a successful model from a similar lesson, written by a student in the previous school year.</td> <td></td> <td></td> </tr> <tr> <td>Ask students to sketch a picture of what their abstract noun might look like if it were a person or creature.</td> <td></td> <td></td> </tr> <tr> <td>Ask students to examine the meaning of death in various cultures and traditions.</td> <td></td> <td></td> </tr> <tr> <td>Ask students to research the meaning of other major life events.</td> <td></td> <td></td> </tr> </tbody> </table>		Effective	Not Effective	Help students brainstorm a list of abstract nouns other than "death" and write the students' suggestions on the board.			Share a successful model from a similar lesson, written by a student in the previous school year.			Ask students to sketch a picture of what their abstract noun might look like if it were a person or creature.			Ask students to examine the meaning of death in various cultures and traditions.			Ask students to research the meaning of other major life events.			<div style="border: 1px solid black; padding: 2px; margin-bottom: 2px;">Assessment Tools</div> <div style="display: flex; justify-content: space-around; font-size: small;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>
	Effective	Not Effective																			
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Ask students to examine the meaning of death in various cultures and traditions.																					
Ask students to research the meaning of other major life events.																					

Figure B16 Question #16.

Task	Question Number		Assessment Tools
MET1	17 of 26	<p><i>Questions 14-17 are based on the following teaching scenario.</i></p> <p>Ms. Garcia is working with an advanced sixth-grade reading group. She is using the following excerpt from <i>Their Eyes Were Watching God</i> by Zora Neale Hurston as a model for a writing assignment.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>So Janie began to think of Death. Death, that strange being with the huge square toes who lived way in the West. The great one who lived in a straight house like a platform without sides to it, and without a roof. What need has Death for a cover, and what winds can blow against him? He stands in his high house that overlooks the world. Stands watchful and motionless all day with his sword drawn back, waiting for the messenger to bid him come. Been standing there before there was a where or a when or a then. She was liable to find a feather from his wings lying in her yard any day now.</p> </div> <p>Latoya is a gifted student who tends to finish her assignments before anyone else in the class. Her work is consistently satisfactory, but Ms. Garcia senses that she is capable of developing her writing even further. Ms. Garcia reads Latoya's first draft.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Fear walks around all by herself          And no one can see her.          She has straight brown hair          And her blue eyes droop above her bags.          She follows person after person,          But the one she is looking for,          She will never find...          Happiness.</p> </div> <p>Which of the following strategies would be the most effective for helping Latoya develop a second draft that mimics Hurston's passage?</p> <ul style="list-style-type: none"> <li><input type="radio"/> Ask Latoya to elaborate by using specific details and action verbs to describe what Fear looks like and how she moves.</li> <li><input type="radio"/> Ask Latoya to list images, animals, or objects that might be used to characterize Fear and Happiness.</li> <li><input type="radio"/> Ask Latoya to read her paragraph aloud and consider how her audience might understand the passage.</li> <li><input type="radio"/> Ask Latoya to reread the excerpt from the novel and compare it to her draft.</li> </ul>	<div style="border: 1px solid black; padding: 2px; margin-bottom: 2px;">Assessment Tools</div> <div style="display: flex; justify-content: space-around; font-size: small;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Figure B17 Question #17.

Task	Question Number	Assessment Tools
MET1	18 of 26	Back Review Next

Ms. Verde is working with her sixth-grade students on persuasive writing. She wants her students to choose a position and then argue either for or against that position. She assigns the topic, "Should students be required to wear school uniforms at school?" Ms. Verde begins the lesson with a whole group activity in which students quickly list as many ideas as they can think of to support both sides of the argument.

Explain how generating ideas for both sides of a topic could support students in writing a persuasive argument.

Figure B18 Question #18.

Task	Question Number	Assessment Tools
MET1	19 of 26	Back Review Next

Ms. Rice begins a unit on memoir writing by reading a passage from a literary model. She then asks students to complete a warm-up activity to help them generate ideas for their own writing.

For each assignment, indicate whether it will help students focus their brainstorming on generating a memoir.

	Will help focus brainstorming	Will not help focus brainstorming
Write a poem about the the ways you have changed, using the form "I used to be...but now I am..."		
Write a sequence of sentences describing some of your experiences, beginning each sentence with the phrase "I remember."		
Write a few adjectives that describe your personality.		
Write down some of your favorite foods and describe what you like about them.		

Figure B19 Question #19.



Task	Question Number	Assessment Tools
MET1	20 of 26	<div style="display: flex; justify-content: flex-end; gap: 5px;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Figueroa and her students are reading an excerpt from the Wallace Stevens poem "Anecdote of the Prince of Peacocks."

I knew the dread  
Of the bushy plain,  
And the beauty  
Of the moonlight  
Falling there,  
Falling  
As sleep falls  
In the innocent air.

After discussing the definition of simile, Ms. Figueroa asks students to write down as many similes as they can think of that describe moonlight. She notices that Regine, a struggling student, isn't writing at all. When Ms. Figueroa asks her why, Regine says, "Writing a simile about moonlight doesn't make sense. Moonlight is just like moonlight; it isn't like anything else."

Which of the following teaching responses is most likely to help Regine write a simile for moonlight?

- ☐ "Of course it isn't *exactly* like anything else. What you need to do is think about a comparison."
- ☐ "What color is the moonlight? Can you think of something else that has the same color?"
- ☐ "The sunlight is a lot like moonlight, isn't it? Except sunlight is warm and moonlight isn't."
- ☐ "Remember last week when we talked about how the sun is a flame? We are doing something similar here."

Figure B20 Question #20.

Task	Question Number	Assessment Tools
MET1	21 of 26	<div style="display: flex; justify-content: flex-end; gap: 5px;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Okeke's fifth-grade class is reading a short story entitled "The Terrible Summer." She wants to use sentences from this story to model how the author uses the technique of "showing" rather than "telling" in her writing.

Which of the following sentences should Ms. Okeke select to best demonstrate the concept of "showing"?

- ☐ The sun flamed like a giant match, and the mercury was hovering at a hundred degrees.
- ☐ It was such a feverish afternoon, everyone felt like they were in an oven.
- ☐ The asphalt bubbled, and ice cream cones melted before the fifth lick.
- ☐ Bill detested the sweltering summer, so he conspired to stay in the delicious coolness of his basement.

Figure B21 Question #21.

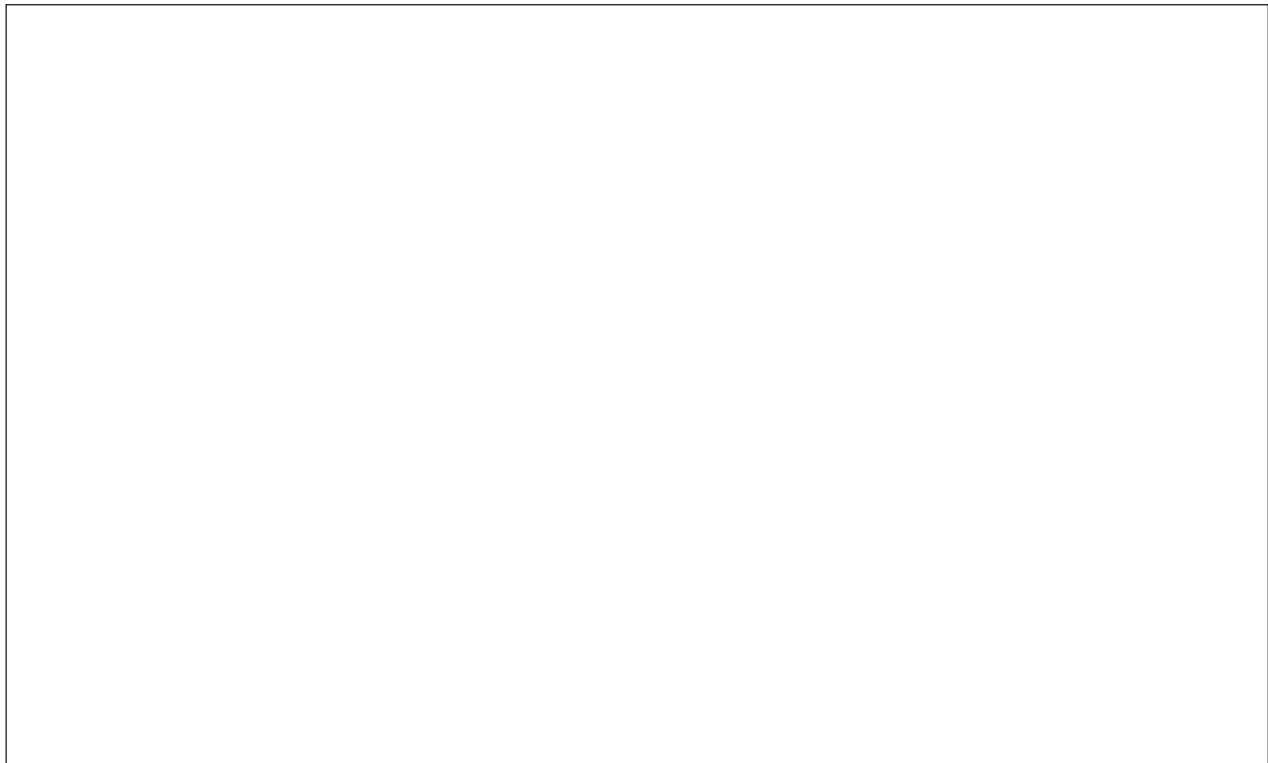


Figure B22 Question #22. (Question #22 in this assessment cannot be shown due to copyright restrictions for an excerpt used from a published text in this question.)

Task	Question Number	Assessment Tools
MET1	23 of 26	Back Review Next

Mr. Jackson's sixth- and seventh-grade students are writing flash fiction. After reading the following draft of Arielle's short story, Mr. Jackson wants to point out some of the strengths in her writing.

Natasha left the house and walked sadly down the sidewalk, slowly shuffling her feet. As she walked by the old house, she was surprised by how sad she felt. She stepped onto the grass, and the sprinklers sprayed her face. The sprinkle of cold water caught her by surprise. Her brother shouted down the street asking her to wait up. Thankfully he couldn't tell the tears from the drops of water.

For each writing feature, indicate whether it describes a strength in Arielle's writing.

	Describes a strength in Arielle's writing	Does not describe a strength in Arielle's writing
Strong character development		
Use of figurative language		
Conveying emotions through showing		
Use of powerful images		

Figure B23 Question #23.

Task	Question Number		Assessment Tools
MET1	24 of 26	<p>Mr. Able's sixth-grade students have just begun writing personal narratives. Manuel produces the following first draft.</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <p>Cars are cool. I collect them. They can open the doors &amp; trunk. I can make a list of the cars I know 1st is a Ferari f50, a Viper GTs, orrr a Mustang GT, Lamberginy, corvette, Lexis GT, 1800 Honda GT, Porsche 911 Torbo, Mercedes, Mini. The Mini is cool. It is small and has mini spinners. I imagine where I could ride in my car. I drive in my Mini all day and go to places MC Donald's any time I want. I love car's there just really special to me. There really cool that's wy I like them. If I'm rich I would buy lots of cars. My favorite car is my limo it light's up it's really cool. It reminds me of my to Disney land.</p> </div> <p>Mr. Able and Manuel have the following conversation during a writing conference.</p> <p><i>Mr. Able:</i> It seems like you're really interested in collecting cars, that you're really passionate about it. How did you get started?</p> <p><i>Manuel:</i> I collect cars because they're cool. I was going to bring my cars in for Expert Day.</p> <p><i>Mr. Able:</i> Let's slow down for a minute. What was the first car you ever collected?</p> <p><i>Manuel:</i> My mom saw a show on Disney channel about kids who collect things.</p> <p><i>Mr. Able:</i> Can you tell me about your first car? Do you remember getting it?</p> <p><i>Manuel:</i> Um.... it was my yellow Corvette.</p> <p><i>Mr. Able:</i> And how did you get your yellow Corvette? Was it a present?</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">             In this writing conference, Mr. Able is attempting to help Manuel improve his narrative by           </div> <ul style="list-style-type: none"> <li><input type="radio"/> focusing his story on a single idea</li> <li><input type="radio"/> developing more vivid descriptions</li> <li><input type="radio"/> including a broader perspective on his topic</li> <li><input type="radio"/> explaining why he likes collecting things</li> </ul>

Figure B24 Question #24.

Task	Question Number		Assessment Tools
MET1	25 of 26	<p>Mr. Blunk's fourth-grade students are learning to write five-paragraph research reports about how Sioux Native Americans lived before the colonial era. Bruce submits the following first draft of an introductory paragraph based on his research.</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <p>The Sioux peoples were very different from us. To live, they needed food, clothing, and shelter. All of their food, clothing, and shelter was different from ours. The Sioux were extremely interesting. In my opinion, everyone should be interested in the Sioux.</p> </div> <p>To help Bruce develop his first paragraph of the research report, which of the following questions would be best for Mr. Blunk to ask during a one-on-one conference?</p> <ul style="list-style-type: none"> <li><input type="radio"/> "What is another word you could use instead of 'interesting?'"</li> <li><input type="radio"/> "How could you vary your sentence structure to improve the flow?"</li> <li><input type="radio"/> "How would you convince someone to agree with your opinion?"</li> <li><input type="radio"/> "What are you going to tell us about their 'food, clothing, and shelter?'"</li> </ul>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">             Assessment Tools           </div> <div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Figure B25 Question #25.

Task	Question Number		Assessment Tools
MET1	26 of 26	<p>Ms. Lee is teaching a unit on persuasion to her sixth- and seventh-grade students. Although the students have read articles on the pros and cons of their topic—eliminating vending machines from school cafeterias—she notices that they have not used textual evidence in their essays.</p> <p>Which of the following is the most effective activity to help Ms. Lee's students add textual evidence to their essays to support their arguments?</p> <p>Ms. Lee should ask her students to</p> <ul style="list-style-type: none"> <li><input type="radio"/> critically read articles about the pros and cons of having vending machines in schools</li> <li><input type="radio"/> record evidence they find in articles on sticky notes and place the sticky notes in the appropriate places in their drafts</li> <li><input type="radio"/> peer edit each others' drafts with a focus on identifying both strengths and weaknesses in the essays</li> <li><input type="radio"/> use their writing guides to apply the proper citation style for authoritative sources</li> </ul>	<div style="border: 1px solid black; padding: 2px; display: inline-block;"> <div style="display: flex; justify-content: space-between; width: 100%;"> <span>Back</span> <span>Review</span> <span>Next</span> </div> </div>

Figure B26 Question #26.

## Appendix C

### Mathematics 6–8 Assessment

Table C1 Item/Question Sequence Number for Mathematics 6–8 Assessment

Item/question sequence number	Answer key	Item/question sequence number	Answer key
1	C	11e	Provides
2	C	11f	Does not provide
3a	Provides	12	B
3b	Provides	13	A
3c	Does not provide	14a	Provides
4	B	14b	Does not provide
5	B	14c	Provides
6a	Provides	14d	Provides
6b	Does not provide	15	B
6c	Provides	16a	Provides
6d	Provides	16b	Provides
6e	Provides	16c	Provides
7	A	16d	Does not provide
8a	Valid	16e	Does not provide
8b	Valid	17	A
8c	Not valid	18	B
8d	Valid	19a	Does not demonstrate
8e	Not valid	19b	Demonstrates
9	D	19c	Demonstrates
10	B	19d	Demonstrates
11a	Provides	20	B
11b	Provides	21	C
11c	Does not provide	22	Open-ended response
11d	Provides	23	Open-ended response

Task	Question Number		Assessment Tools
MET	1 of 23	<p>Ms. Hupman is teaching an introductory lesson on exponents. She wants to give her students a quick problem at the end of class to check their proficiency in evaluating simple exponential expressions. Of the following expressions, which would be <u>least</u> useful in assessing student proficiency in evaluating simple exponential expressions?</p> <p> <input type="radio"/> <math>3^3</math>  <input type="radio"/> <math>2^3</math>  <input type="radio"/> <math>2^2</math>  <input type="radio"/> All of these are equally useful in assessing student proficiency in evaluating simple exponential expressions.         </p>	<div style="border: 1px solid black; padding: 2px;">           Back   Review   Next         </div>

Figure C1 Question #1.

Task	Question Number		Assessment Tools
MET	2 of 23	<p>In a unit introducing proportions, Mr. Hayes gave his students the following problem.</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <p>Sabrina's Flower Shop has 24 roses and 36 carnations. What is the ratio of the number of roses to the total number of roses and carnations at Sabrina's Flower Shop?</p> </div> <p>When the students finished, he wanted to give them a similar practice problem. Of the following, which asks students to do the most similar mathematical work and thinking as the problem above?</p> <p> <input type="radio"/> A spinner has 10 equal sections numbered 1 through 10. What are the odds that the spinner will land on a prime number when it is spun?  <input type="radio"/> Nick's Team Sports Store has 24 basketballs and 60 footballs on display. What is the ratio of the number of basketballs to the number of footballs on display?  <input type="radio"/> For breakfast during the month of April, Katherine ate cereal on 18 days and eggs on the other 12 days. What is the ratio of the number of days Katherine ate cereal for breakfast to the number of days in April?  <input type="radio"/> All of these problems ask students to do the same mathematical work and thinking.         </p>	<div style="border: 1px solid black; padding: 2px;">           Back   Review   Next         </div>

Figure C2 Question #2.

Task	Question Number		Assessment Tools												
MET	3 of 23	<p>During an introductory lesson on solving equations, Mr. Chen gave his students the following problem.</p> $10 - \square = 12 - 7$ <p>He asked his students to decide what number can be put into the box to make the sentence true. Students got the correct answer of 5, but they used different strategies. For each of the following strategies, indicate whether or not it provides evidence of mathematically valid reasoning about this problem.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 60%;"></th> <th style="width: 20%; text-align: center;">Provides Evidence of Mathematically Valid Reasoning</th> <th style="width: 20%; text-align: center;">Does Not Provide Evidence of Mathematically Valid Reasoning</th> </tr> </thead> <tbody> <tr> <td>12 - 7 = 5, so I need to find out what I take away from 10 to get 5. 10 - 5 = 5, so the answer is 5.</td> <td></td> <td></td> </tr> <tr> <td>10 is two less than the 12 on the other side, so I need to take away two from 7 to get the same answer of 5.</td> <td></td> <td></td> </tr> <tr> <td>When I subtract 12 minus 7 the answer I get is 5, so 5 goes into the box to make the sentence true.</td> <td></td> <td></td> </tr> </tbody> </table>		Provides Evidence of Mathematically Valid Reasoning	Does Not Provide Evidence of Mathematically Valid Reasoning	12 - 7 = 5, so I need to find out what I take away from 10 to get 5. 10 - 5 = 5, so the answer is 5.			10 is two less than the 12 on the other side, so I need to take away two from 7 to get the same answer of 5.			When I subtract 12 minus 7 the answer I get is 5, so 5 goes into the box to make the sentence true.			<div style="border: 1px solid black; padding: 2px;"> <span style="border: 1px solid black; padding: 0 5px;">Back</span> <span style="border: 1px solid black; padding: 0 5px;">Review</span> <span style="border: 1px solid black; padding: 0 5px;">Next</span> </div>
	Provides Evidence of Mathematically Valid Reasoning	Does Not Provide Evidence of Mathematically Valid Reasoning													
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10 is two less than the 12 on the other side, so I need to take away two from 7 to get the same answer of 5.															
When I subtract 12 minus 7 the answer I get is 5, so 5 goes into the box to make the sentence true.															

Figure C3 Question #3.

Task	Question Number		Assessment Tools
MET	4 of 23	<p>At the start of a lesson on finding the side length of a square given its area, Ms. Sabatine gave her students a problem to assess their prior knowledge. Several students incorrectly answered that the side length of a square with area 36 square units is 9 units. At the end of the lesson, Ms. Sabatine wanted to give a similar problem to assess what her students had learned. Of the following areas to use in this problem, which would be <u>least</u> useful for assessing student learning in this situation?</p> <ul style="list-style-type: none"> <li><input type="radio"/> 9 square units</li> <li><input type="radio"/> 16 square units</li> <li><input type="radio"/> 100 square units</li> <li><input type="radio"/> All of these are equally useful for assessing student learning in this situation.</li> </ul>	<div style="border: 1px solid black; padding: 2px;"> <span style="border: 1px solid black; padding: 0 5px;">Back</span> <span style="border: 1px solid black; padding: 0 5px;">Review</span> <span style="border: 1px solid black; padding: 0 5px;">Next</span> </div>

Figure C4 Question #4.



Task	Question Number		Assessment Tools
MET	5 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

To assess her students' prior knowledge about evaluating arithmetic expressions, Ms. Santiago assigned a worksheet of problems. She noticed that Alexis answered the first two incorrectly and the next two correctly.

1)  $7 \times 2 - 6 + 3 = 5$

2)  $9 - 5 + (16 \div 8) = 2$

3)  $9 + 24 + 3 - 1 = 16$

4)  $17 - (3 + 7 \times 2) = 0$

Which of the remaining problems is Alexis likely to answer incorrectly?

☐  $8 + 7 - 12 \div 3$   
☐  $13 - 3 \times 2 + 5$   
☐  $(27 \div 3 - 4) + 8$   
☐  $(16 - 12) \times 5 + 10$

Figure C5 Question #5.

Task	Question Number		Assessment Tools
MET	6 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

In a unit on proportional reasoning, Ms. Richmond's class was discussing the following problem.

If 4 cups of cocoa and 2 cups of sugar yield 16 brownies, how many cups of cocoa and how many cups of sugar are needed to make 24 brownies?

Ms. Richmond's students used different strategies to solve the problem. Of the following, which provides evidence of mathematically valid student thinking? For each strategy, indicate whether or not it provides evidence of mathematically valid student thinking.

	Provides Evidence of Mathematically Valid Student Thinking	Does Not Provide Evidence of Mathematically Valid Student Thinking
48 brownies need 12 cups of cocoa and 6 cups of sugar. To make 24 brownies, I need 6 cups of cocoa and 3 cups of sugar.		
4 and 2 both go into 16. 4 plus 2 is 6, half of 6 is 3, and 6 and 3 both go into 24, so you need 6 cups of cocoa and 3 cups of sugar to make 24 brownies.		
1 brownie needs $\frac{1}{4}$ cup of cocoa and $\frac{1}{8}$ cup of sugar. To make 24 brownies, I need to multiply by 24 for cocoa and sugar. Thus, I need 6 cups of cocoa and 3 cups of sugar.		
6 cups of cocoa and sugar makes 16 brownies, so 24 brownies need 9 cups of cocoa and sugar. Since the ratio of cocoa to sugar is 2:1, I need 6 cups of cocoa and 3 cups of sugar.		
Since 1 cup of sugar is needed to make 8 brownies, I need 3 cups of sugar to make 24 brownies. The amount of cocoa is two times the amount of sugar in the recipe, so I need 6 cups of cocoa.		

Figure C6 Question #6.

Task	Question Number		Assessment Tools
MET	7 of 23		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Mr. Philbey gave his students the following fraction multiplication problem to evaluate.

$$\frac{15}{8} \times \frac{36}{5} \times \frac{14}{3} \times \frac{1}{6} \times \frac{5}{12} \times \frac{2}{7} =$$

Several of his students agreed that the answer is  $\frac{5}{4}$ , but their approaches to the problem were very different. Of the following student strategies, which would not be valid for solving this problem?

☐ You can cancel the 5s and the 36s and the 2s, then you get  $1 \times 14 \times 1 \times 5$  over  $8 \times 1 \times 7$ , which is 70 over 56, which simplifies to  $\frac{10}{8}$  or just  $\frac{5}{4}$ .

$$\frac{1\cancel{5}}{8} \times \frac{\cancel{3}6}{\cancel{5}} \times \frac{14}{\cancel{3}} \times \frac{1}{\cancel{6}} \times \frac{5}{12} \times \frac{2}{7} = \frac{1 \times 14 \times 1 \times 5}{8 \times 1 \times 7} = \frac{70}{56} = \frac{10}{8} = \frac{5}{4}$$

☐ You can factor and just get 5 divided by 4.

$$\frac{15}{8} \times \frac{36}{5} \times \frac{14}{3} \times \frac{1}{6} \times \frac{5}{12} \times \frac{2}{7} = \frac{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{7} \times 5 \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{7}} = \frac{5}{4}$$

☐ You can cancel the 15 and the 5 and 3, and then cancel the 6 and 12 with the 36 and the 2 from the 14. Then you can cancel the 7s, and you get 10 over 8, or  $\frac{5}{4}$ .

☐ All of these are valid strategies for evaluating this expression.

Figure C7 Question #7.

Task	Question Number	Assessment Tools
MET	8 of 23	Back Review Next

During a lesson on solving multistep equations, Mr. Steinbrecher asked his students to solve the equation  $4(5x - 11) = 16$ . While walking around the class looking at what the students were writing, he noticed several different strategies. For each of the following student solutions, indicate whether or not it is a valid strategy for solving this problem.

	Strategy Is Valid	Strategy Is Not Valid
$\frac{1}{4} \cdot 4(5x - 11) = 16 \cdot \frac{1}{4}$ $5x - 11 = 4$ $5x = 15$ $x = 3$		
$4 \cdot 4(5x - 11) = 16 \cdot 4$ $\frac{16(5x - 11)}{16} = \frac{16 \cdot 4}{16}$ $5x - 11 = 4$ $\begin{array}{r} +11 \\ +11 \end{array}$ $5x = 15$ $x = 3$		
$4(5x - 11) = 16$ $9x - 11 = 16$ $\begin{array}{r} +11 \\ +11 \end{array}$ $\frac{9x}{9} = \frac{27}{9}$ $x = 3$		
$4(5x - 11) = 16$ $\frac{20x - 44}{20} = \frac{16}{20}$ $\begin{array}{r} + \frac{44}{20} \\ + \frac{44}{20} \end{array}$ $x = \frac{60}{20}$ $x = 3$		
$\frac{4}{4} \cdot \frac{5x - 11}{4} = \frac{16}{4}$ $\frac{5x}{4} - \frac{11}{4} = 4$ $\begin{array}{r} + \frac{11}{4} \\ + \frac{11}{4} \end{array}$ $\frac{5x}{5} \cdot \frac{4}{4} = \frac{18}{4} \cdot \frac{4}{4}$ $x = 3$		

Figure C8 Question #8.

Task	Question Number	Assessment Tools
MET	9 of 23	<div style="display: flex; gap: 5px;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>


Ms. Lowe is teaching a unit on statistics. Some of the students in her class speak a language other than English at home and when they work in small groups. As she prepares for the unit, she makes a note of vocabulary words that she believes will be challenging for her students, especially words that have different meanings in different contexts. Of the following vocabulary words that she identified, which will require the least clarification regarding differences in meaning?

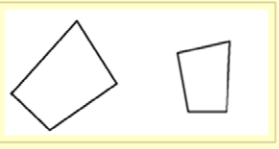
- ☐ Mean
- ☐ Mode
- ☐ Range
- ☐ Quartile


Figure C9 Question #9.

Task	Question Number	Assessment Tools
MET	10 of 23	<div style="display: flex; gap: 5px;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Mr. Cuomo's students have had trouble finding corresponding sides of similar figures. After spending a class period working on this issue, Mr. Cuomo wanted to assess if his students could find the corresponding sides of similar figures. Which pair of similar figures is the best choice for this assessment purpose?

☐


☐


☐


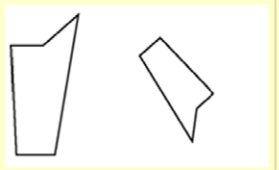
☐


Figure C10 Question #10.

Task	Question Number		Assessment Tools
MET	11 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

During a unit on proportional reasoning, Ms. Tran gave her students the following problem.

Two jugs each have the same amount of fruit punch. Jug #1 has 5 ounces of fruit punch concentrate for every 9 ounces of water. Jug #2 has 11 ounces of fruit punch concentrate for every 14 ounces of water. Which jug contains the fruit punch with the stronger taste, or will the fruit punch in both jugs taste the same?

Students got the correct answer that Jug #2 contains the fruit punch with the stronger taste, but they thought about the problem in different ways. For each of the following student explanations, indicate whether or not it provides evidence of mathematically valid reasoning about this problem.

	Provides Evidence of Mathematically Valid Reasoning	Does Not Provide Evidence of Mathematically Valid Reasoning
Jug #1 has $1\frac{4}{5}$ ounces of water for each ounce of concentrate. Jug #2 only has $1\frac{3}{11}$ ounces of water for each ounce of concentrate, so Jug #2 is stronger.		
11 times the amounts in Jug #1 gives 55 ounces of concentrate and 99 ounces of water. 5 times the amounts in Jug #2 gives 55 ounces of concentrate and 70 ounces of water. When the concentrate is the same, there is more water in Jug #1, so Jug #2 is stronger.		
$\frac{11-5}{14-5} = \frac{6}{9}$ , and the ratio of concentrate to water in Jug #1 is only 5 parts water to 9 parts concentrate, so Jug #2 is stronger.		
I looked at the ratios of concentrate to water. For Jug #1, $\frac{5}{9} < \frac{6}{9} = \frac{2}{3}$ , and for Jug #2, $\frac{11}{14} > \frac{11}{15} > \frac{10}{15} = \frac{2}{3}$ , so Jug #2 is stronger.		
In Jug #1, the concentrate is less than 60% of the water. In Jug #2, the concentrate is more than 75% of the water, so Jug #2 is stronger.		
If you add 6 to both of the numbers for Jug #1, you get 11 ounces of concentrate and 15 ounces of water. This is more water than what is in Jug #2 for the same amount of concentrate, so Jug #2 is stronger.		

Figure C11 Question #11.

Task	Question Number		Assessment Tools
MET	12 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Seidel is introducing the distributive property. To motivate her students, she wants to give them an example that will focus their attention on how using the distributive property can simplify computations. In which of the following examples will the use of the distributive property most simplify the computations?

☐  $12 \times 29 + 12 \times 38 = \underline{\hspace{2cm}}$   
☐  $17 \times 37 + 17 \times 63 = \underline{\hspace{2cm}}$   
☐  $13 \times 13 + 15 \times 15 = \underline{\hspace{2cm}}$   
☐  $16 \times 24 + 16 \times 24 = \underline{\hspace{2cm}}$

Figure C12 Question #12.

Task	Question Number		Assessment Tools
MET	13 of 23		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Malloy's textbook suggests using white chips (for positive integers) and red chips (for negative integers) to teach integer subtraction. To solve the problem  $2 - (-3)$ , the book describes the process below.

1. Start with two white chips.
2. Add three pairs of red and white chips.
3. Then take away three red chips for the  $-3$ .
4. There are 5 white chips left, so the answer is 5.

Ms. Malloy is concerned that her students will see that adding red and white chips sets up the subtraction, but not understand why the additional chips can be added. Of the following, which provides a mathematical explanation for why the three pairs of red and white chips can be added?

☐ By adding both a red chip and a white chip to an expression, you are really adding a zero, which does not change the value of the original amount.

☐ The 2 is two white chips and the  $-3$  is three red chips. When you add the three red chips, you also add three white chips. That way, when you take away the red chips, the total number of white chips will be 5, which is the correct answer.

☐ The three red chips are for the  $-3$  and the three white chips are what the red chips become because the negative of a negative is positive.

☐ Red chips represent negative integers, but when you subtract negative three, you get positive three, so you need to add three white chips as well as three red chips.

Figure C13 Question #13.

Task	Question Number		Assessment Tools
MET	14 of 23		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Spiess's class had just finished a unit on operations with fractions. Before moving on to the next unit, she wanted to see if her students understood the invert-and-multiply rule for dividing fractions.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Ms. Spiess asked her students to explain why the algorithm works. For each of the following explanations, indicate whether or not it provides evidence that the student understands why the algorithm works.

	Provides Evidence	Does Not Provide Evidence
It's because you can think of it as a fraction and just multiply by one to get rid of the denominator. $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \times \frac{d}{d}}{\frac{c}{d} \times \frac{d}{d}} = \frac{a}{b} \times \frac{d}{c}$		
Dividing is like cross-multiplying, so you need to multiply the numerator of the first with the denominator of the second and then the denominator of the first with the numerator of the second. That gives you $\frac{ad}{bc}$ .		
When you find a common denominator you get $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bd} \div \frac{bc}{bd}$ , but if you want to know how many $\frac{bc}{bd}$ go into $\frac{ad}{bd}$ that's just the same as how many $bc$ go into $ad$ . So, it's $\frac{ad}{bc}$ .		
I can see that $\frac{ad}{bc} \times \frac{c}{d} = \frac{a}{b}$ , so when you divide by $\frac{c}{d}$ you get $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ .		

Figure C14 Question #14.



Task	Question Number		Assessment Tools
MET	15 of 23		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Jones's class has been studying area and perimeter. Some of the students in her class are learning English; they speak Spanish at home and when they work in small groups. One day they work on the following problem.

- Look for all of the rectangles with area 36 (where length and width are integers). Write down the dimensions.
- Calculate the perimeter for each rectangle.
- Describe a pattern relating the perimeter and the dimensions.

Of the following student responses, which most accurately describes the relationship between the dimensions of a rectangle and its perimeter?

☐ A rectangle has ... two ... short sides, and two long sides. The perimeter of the ... long ... skinny rectangle is big. It's about twice longer than the square.

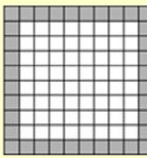
☐ The longer the ah ... the longer [traces the shape of a long rectangle with her hands several times] the ah ... the longer the, thing, you know the more the perimeter, the higher the perimeter is.

☐ The rectangle ... the one with 4 width and 9 length ... its perimeter 26, and its area is bigger, it's 36. The rectangle 3 width and 12 length ... it's perimeter 30. It's 4 more ... it goes up by 4 ... higher.

☐ Each of these student responses describes the relationship with equal accuracy.

Figure C15 Question #15.

Task	Question Number		Assessment Tools
MET	16 of 23		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>



Ms. Kamp asks her students to consider squares of different side lengths with only the boxes along the sides shaded as in the figure on the left.

She asks each student to write an expression for the number of shaded boxes in a square with a side length of  $n$  boxes and to explain why the expression gives the number of shaded boxes for any size square. For each of the following explanations, indicate whether or not it provides evidence that the student understands why the expression can be used to find the number of such shaded boxes in any square.

	Provides Evidence	Does Not Provide Evidence
If you start on the bottom left and go to just below the top left, then start with the top left and go to just before the top right, and keep doing that, you will get 4 groups, and each group has 1 less than $n$ , so you get $4(n-1)$ .		
My expression is $n + 2(n-1) + (n-2)$ because the top of the square has $n$ shaded boxes, then each of the sides has $n-1$ shaded boxes left, and then the bottom has $n-2$ shaded boxes left.		
Inside the square with a side length of $n$ boxes is a square with side length of $n-2$ boxes, so if you find the area of the two squares and subtract them, you will find the number of shaded boxes. So, I get $n^2 - (n-2)^2$ .		
I get $4(n-2) + 4$ because there are 36 boxes shaded, and when you put 10 in for the $n$ in $4(n-2) + 4$ and follow the order of operations, the answer is 36.		
My expression is $2n + 2(n-2)$ because if you simplify $2n + 2(n-2)$ you get $2n + 2n - 4$ , which is equal to $4n - 4$ , and because this doesn't depend on $n$ , it works for any $n$ .		

Figure C16 Question #16.

Task	Question Number		Assessment Tools
MET	17 of 23		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Jensen had been working on rational numbers with her students for some time, and she wanted them to understand that between any two rational numbers, one can always find another rational number. To support this thinking, she asked her students to first think about how to find a fraction between any two positive fractions, such as  $\frac{1}{3}$  and  $\frac{1}{2}$ . As the class worked on the problem, one student said, "I think you can find a fraction between any two positive fractions by adding the denominators and numerators—here you just get  $\frac{1+1}{3+2} = \frac{2}{5}$ , which is greater than  $\frac{1}{3}$  and less than  $\frac{1}{2}$ ."

Of the following descriptions, which best characterizes the student's technique for finding a number between any two positive fractions?

- ☐ The technique is a simple method that works for any two positive fractions.
- ☐ The technique only works for some positive fractions.
- ☐ The technique produces messy answers in many cases and should be avoided.
- ☐ The technique should have been to find the average of the two fractions, i.e.,  $\frac{1}{2} \left( \frac{1}{3} + \frac{1}{2} \right) = \frac{1}{2} \left( \frac{2}{6} + \frac{3}{6} \right) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$ .

Figure C17 Question #17.

Task	Question Number		Assessment Tools
MET	18 of 23		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Delgado's students are reasoning about whether or not a given number is divisible by 3. She is concerned that even though her students do well listing multiples of numbers, they have not established the general property that if a number is a multiple, then when it is written as a sum of two numbers, both must be multiples or neither is a multiple. She recognizes that additional work is needed for the idea to be meaningful to the whole class.

Of the following student explanations for why 80 is not divisible by 3, which uses this general property?

- ☐ If I have one more, it is a multiple of 9. So, 80 is not divisible by 3.
- ☐ Every 10 leaves 1 remainder, so 80 has 8 remainders. You can't divide 8 into 3 equal numbers, so 80 is not divisible by 3.
- ☐ First, 80 is divisible by 2. If I divide by 2 again and again, I got 40, 20, 10, and 5. I can't find 3 anywhere, so it is not divisible by 3.
- ☐ Each of these explanations uses this general property.

Figure C18 Question #18.

Task	Question Number	Assessment Tools
MET	19 of 23	Back Review Next

Mr. Whalen wants to review strategies for finding the greatest common factor of two numbers with his students. At the beginning of class, he asks his students to find the greatest common factor of 180 and 84 and to explain how they found it. While walking around the class looking at what the students are writing, he notices several different strategies. For each of the following responses, indicate whether or not it demonstrates a valid strategy for finding the greatest common factor of any two numbers.

	Demonstrates a Valid Strategy	Does Not Demonstrate a Valid Strategy
<p>180 = 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90, 180</p> <p><del>84 = 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84</del></p> <p>I listed all the numbers that go into 180 and then looked to see what goes into 84. 20, 15, and 9 didn't work, but 12 did.</p>		
<p>180: 2, 3, 5, 9, 15, 45, 90, 180</p> <p>84: 2, 3, 7, 12, 21, 42, 84</p> <p>GCF = 2 · 2 · 3 = 12</p> <p>I used prime factor trees and looked for the common factors. Both had two 2s and a 3, so I multiplied.</p>		
<p>4   180 84</p> <p>3   45 21</p> <p>15 7</p> <p>GCF = 4 · 3 = 12</p> <p>I saw that 4 goes into both and that 3 goes into 45 and 21. Nothing goes into 15 and 7.</p>		
<p>15 7</p> <p>45 21</p> <p>90 42</p> <p>180 84</p> <p>2 · 2 · 3 2 · 2 · 3</p> <p>GCF = 2 · 2 · 3 = 12</p> <p>I tried different numbers. I divided both, and then another 2 and then a 3. There isn't anything for 15 and 7, so the answer is 12.</p>		

Figure C19 Question #19.

Task	Question Number		Assessment Tools
MET	20 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Lang's class had been studying the concept of slopes of lines, so she asked them to consider all of the lines passing through one point and how the slopes of those lines vary. The students had used geoboards in some earlier work, so they started talking about the slopes of lines on an "infinitely extended" geoboard. (Geoboards are flat blocks of wood, roughly one foot square, with pegs laid out on a grid where rubber bands can be hooked to make lines or polygons.) The students decided that the pegs of the infinite geoboard could be thought of as the set of points with integer coordinates in the Cartesian plane.

During the discussion, students had the following exchange.

Yonah: On the geoboard, you can't get all of the slopes, because the geoboard points are too spread out—there are a whole bunch of lines between the ones you can make.

Andy: I disagree. I think we can make any slope. Starting at one point, by choosing another geoboard point far enough away, we can tilt the line as much or as little as we like.

Becky: What I was thinking was if you run a line through one geoboard point, it will always hit another one far enough out.

Of the following concepts, which is most directly related to the mathematics underlying this discussion?

- ☐ Interpretation of the derivative—the derivative is the slope of the tangent line.
- ☐ Density of numbers on the real line—the rational numbers are dense, but not every real number is rational.
- ☐ The parallel postulate—given a point and a line, there is a unique line through the given point parallel to the given line.
- ☐ Each of these concepts is equally related to the underlying mathematics.

Figure C20 Question #20.

Task	Question Number		Assessment Tools
MET	21 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Mr. Evans asked his students to compare  $\frac{13}{15}$  to  $\frac{15}{18}$  and decide which is greater. During the class discussion, Wanda explained her idea.

It's like if our class had 13 boys and 15 girls and then we got 2 more boys and 3 more girls. But  $\frac{2}{3}$  is less than  $\frac{13}{15}$ . If you add 2 to the top and 3 to the bottom it's  $\frac{15}{18}$ , so then  $\frac{15}{18}$  has to be less than  $\frac{13}{15}$ .

Of the following, which best characterizes Wanda's reasoning?

- ☐ Her reasoning is incorrect, because you can't just add numerators and add denominators like she did.
- ☐ Her reasoning is incorrect, because she needs to compare  $\frac{13}{15}$  and  $\frac{15}{18}$  directly.
- ☐ Her reasoning is correct, but it depends on it being easy to compare the ratio of the differences to one of the other ratios.
- ☐ Her reasoning is correct, but it does not work in general—for instance, it wouldn't work if the ratio of the added boys to the added girls were greater than  $\frac{13}{15}$ , such as  $\frac{9}{10}$ .

Figure C21 Question #21.

Task	Question Number		Assessment Tools
MET	22 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Mr. Yusuf is teaching a lesson on extending patterns. He plans to show his students the two patterns below in which each figure is constructed with toothpicks. Each pattern shows the first three figures in the pattern. He will ask his students to write an algebraic expression to represent the number of toothpicks in the  $n$ th figure of each pattern.

Pattern 1:

Pattern 2:

Mr. Yusuf wants to begin with the pattern that is mathematically easier and then move to the pattern that is more complex.

Which pattern should Mr. Yusuf give to his students first?

☐ Pattern 1  
☐ Pattern 2

Explain why the pattern you chose is the mathematically easier pattern of the two patterns given.

Figure C22 Question #22.

Task	Question Number		Assessment Tools
MET	23 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

A lesson in Ms. Hagerman's textbook defines the distributive property, but the exercises merely ask for its definition. To motivate her students to learn the definition, Ms. Hagerman wants to give an example that will illustrate the usefulness of the distributive property. She notices the following four examples in the teacher edition of the textbook.

1.  $12 \times \left(\frac{3}{4} + \frac{1}{4}\right)$

2.  $18 \times \left(\frac{3}{5} - \frac{1}{10}\right)$

3.  $29 \times \left(\frac{5}{6} - \frac{3}{8}\right)$

4.  $36 \times \left(\frac{5}{12} + \frac{2}{9}\right)$

Ms. Hagerman decides that example number 4 would be the best for her to use of the four examples.

Explain why example number 4 is the best example for Ms. Hagerman to use for illustrating the usefulness of the distributive property.

Explain why the other examples do not illustrate the usefulness of the distributive property as well as example number 4 does.

Figure C23 Question #23.



## Appendix D

## English Language Arts (ELA) 7–9 Assessment

Table D1 Item/Question Sequence Number for English Language Arts (ELA) 7–9 Assessment

Item/question sequence number	Answer key	Item/question sequence number	Answer key
1	A	15	Lea, Akeem, Sarah
2	A	16a	Literal
3a	Not likely	16b	Interpretive
3b	Not likely	16c	Personal
3c	Likely	17	A
3d	Not likely	18	Open-ended response
4	B	19a	Will
5	A	19b	Will
6	B	19c	Will not
7a	Should	19d	Will not
7b	Should not	20	B
7c	Should	21a	Has
7d	Should not	21b	Has not
8	D	21c	Has
9	C	22	D
10	D	23a	Does not describe
11a	Accurate	23b	Does not describe
11b	Accurate	23c	Describes
11c	Not accurate	23d	Describes
11d	Accurate	24a	Likely
11e	Not accurate	24b	Not likely
11f	Not accurate	24c	Likely
12	Open-ended response	24d	Likely
13	D	25	A
14	B	26	B

Task	Question Number	Assessment Tools
MET1	1 of 26	Back Review Next

Mr. Hernandez is considering ways to teach his students to identify text structures prior to reading informational text.

Which of the following would be the best activity for this purpose?

☐ Scanning the text for the types of transition words used in the text passage  
☐ Previewing the length of the text passage to see how many subtitles it contains  
☐ Scanning the pictures and captions that accompany the text passage  
☐ Making a K-W-L chart to activate prior knowledge on the topic

Figure D1 Question #1.



Task	Question Number		Assessment Tools
MET1	2 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Ms. Reinhardt has her sixth- and seventh-grade students read the following poem by Emily Dickinson.

**I'm nobody! Who are you?**

I'm nobody! Who are you?  
 Are you nobody, too?  
 Then there's a pair of us—don't tell!  
 They'd banish us, you know.

How dreary to be somebody!  
 How public, like a frog  
 To tell your name the livelong day  
 To an admiring bog!

To make the poem's meaning more accessible, Ms. Reinhardt wants to relate the poem to a topic of interest to her students. She decides to focus on teen celebrities, a current hot topic in her classroom.

Which of the following is the best option to help students use their knowledge about teen celebrities to understand this poem?

Ms. Reinhardt should have students think about the way teen celebrities are

- ☐ often on display, with the media focusing on their every move
- ☐ admired by the public regardless of their positive or negative contributions
- ☐ celebrated in the media for a short time but then hardly thought about
- ☐ sometimes immature and therefore not able to handle the pressures of fame

Figure D2 Question #2. *I'm Nobody* by Emily Dickinson. Reprinted by permission of Dover Publications.

Task	Question Number		Assessment Tools
MET1	3 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Mr. Allen is working with a class of English-language learners. Although his students have become more proficient at conversing in English, they still often have trouble comprehending written text. He is considering possible reasons that might explain why his students could have difficulty comprehending the following two sentences.

Yesterday I went to the opera. The soprano's singing was superb.

For each possible reason, indicate whether it is a likely source of confusion for Mr. Allen's English-language learners.

	Likely source of confusion	Not a likely source of confusion
Difficulty understanding the word "singing"		
Lack of firsthand experience with opera		
Not knowing that "soprano" is an element of the opera		
Misunderstanding the use of alliteration		

Figure D3 Question #3.

Task	Question Number	Assessment Tools
MET1	4 of 26	<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

*Questions 4-5 are based on the following teaching scenario.*

Mr. Goldberg is teaching the science fiction novel *The Giver*, by Lois Lowry, to his sixth- and seventh-grade students. The people in the society presented in the novel have no knowledge of the world outside their community. Only one person in the society, known as "the Giver," holds knowledge of the rest of world. The main character in the novel, Jonas, is designated as the Giver at age 12. In the passage below, Jonas is startled by the look of a newborn baby's eyes. Mr. Goldberg reads the passage aloud to the class:

But he had been startled by the newchild's eyes. Mirrors were rare in the community; they weren't forbidden, but there was no real need of them, and Jonas had simply never bothered to look at himself very often even when he found himself in a location where a mirror existed. Now, seeing the newchild and its expression, he was reminded that the light eyes were not only a rarity but gave the one who had them a certain look—what was it? *Depth*, he decided; as if one were looking into the clear water of the river, down to the bottom, where things might lurk which hadn't been discovered yet. He felt self-conscious, realizing that he, too, had that look.

Mr. Goldberg stops reading when he realizes that some students seem confused.

Which of the following activities would best help Mr. Goldberg's students comprehend the passage?

- ☐ Visualize Jonas and the "newchild" looking at each other.
- ☐ Use evidence from the text to show how Jonas was feeling.
- ☐ Focus on Lowry's use of italics to emphasize certain words.
- ☐ Describe a time when you felt self-conscious like Jonas.

Figure D4 Question #4.

Task	Question Number	Assessment Tools
MET1	5 of 26	<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

*Questions 4-5 are based on the following teaching scenario.*

Mr. Goldberg is teaching the science fiction novel *The Giver*, by Lois Lowry, to his sixth- and seventh-grade students. The people in the society presented in the novel have no knowledge of the world outside their community. Only one person in the society, known as "the Giver," holds knowledge of the rest of world. The main character in the novel, Jonas, is designated as the Giver at age 12. In the passage below, Jonas is startled by the look of a newborn baby's eyes. Mr. Goldberg reads the passage aloud to the class:

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Which of the following questions would best focus a class discussion on the theme of the passage?

- ☐ Why is Jonas startled when he looks into the "newchild's" eyes?
- ☐ Why is Jonas chosen to be the Giver?
- ☐ Why are there so few mirrors in the community?
- ☐ Why has Jonas never bothered to look at himself in a mirror?

Figure D5 Question #5.

Task	Question Number		Assessment Tools
MET1	6 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

As part of a unit on *Romeo and Juliet*, Ms. Bergan wants to work with her students to develop their understanding of plot structure. The curriculum materials Ms. Bergan is using suggest that she introduce the following plot diagram from a textbook to help her students understand the plot structure of *Romeo and Juliet*.



When she introduces this diagram, her students share different ideas about whether it does in fact represent the plot of *Romeo and Juliet*.

*Antonio*: I think that the climax happens in the middle when Tybalt dies because after that everything is downhill.

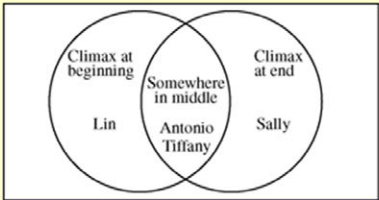
*Sally*: Isn't the climax of *Romeo and Juliet* at the very end when they die and shouldn't the resolution part of the diagram be a lot shorter?

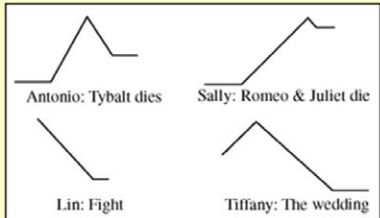
*Lin*: I think the climax is the fight at the beginning and the rest of the story is all a big long resolution of that feud.

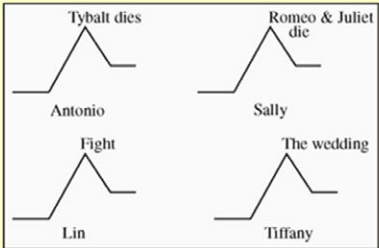
*Tiffany*: I think the climax comes near the beginning with the wedding because it is a love story and that is the most important part of the play.

Ms. Bergan decides to represent students' ideas on the chalkboard in order to begin a discussion of the various interpretations of the climax of *Romeo and Juliet*.

Which of these is an accurate representation of the students' ideas about the climax of *Romeo and Juliet*?

☐ 

☐ 

☐ 

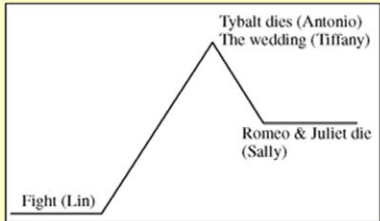
☐ 

Figure D6 Question #6.

Task	Question Number		Assessment Tools
MET1	7 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 7-9 are based on the following teaching scenario.*

Mr. Liu is teaching Harper Lee's *To Kill a Mockingbird* to his eighth-grade English class. The novel is set in a small town in Alabama in the 1930s and is narrated by Scout, an outspoken young girl who is the daughter of attorney Atticus Finch. A major conflict is introduced when Atticus defends a black man, Tom Robinson, who has been wrongly accused of assaulting a white woman. As Atticus works to prove Tom's innocence, Scout and her brother Jem experience harassment at the hands of various townspeople.

Mr. Liu is aware of the difficulties his students are likely to encounter while trying to comprehend these aspects of the story. He is considering topics to include in his instruction in order to help his students better understand the conflict in the story.

For each topic, indicate whether Mr. Liu should include it in his instruction.

	Should include	Should not include
Why a black man might not have been adequately represented in the legal system of the time		
How southern authors like Harper Lee help us to understand our history		
How social attitudes are reinforced through teasing and bullying		
How, even today, people speak badly of lawyers		

Figure D7 Question #7.

Task	Question Number		Assessment Tools
MET1	8 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 7-9 are based on the following teaching scenario.*

Mr. Liu is teaching Harper Lee's *To Kill a Mockingbird* to his eighth-grade English class. The novel is set in a small town in Alabama in the 1930s and is narrated by Scout, an outspoken young girl who is the daughter of attorney Atticus Finch. A major conflict is introduced when Atticus defends a black man, Tom Robinson, who has been wrongly accused of assaulting a white woman. As Atticus works to prove Tom's innocence, Scout and her brother Jem experience harassment at the hands of various townspeople.

Mr. Liu realizes that his students are focusing on nine-year-old Scout as just another character in the novel. He wants his students to focus on Scout as the narrator.

Which of the following questions will best help Mr. Liu's students focus on Scout as the narrator of the novel?

- ☐ How is Scout's behavior unusual for a girl of her social standing in the 1930s South?
- ☐ How does Scout's relationship with Atticus develop during the novel?
- ☐ How does Scout change over the course of the novel?
- ☐ How does Scout's age influence what she sees and what adults tell her about these events?

Figure D8 Question #8.



Task	Question Number		Assessment Tools
MET1	9 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 7-9 are based on the following teaching scenario.*

Mr. Liu is teaching Harper Lee's *To Kill a Mockingbird* to his eighth-grade English class. The novel is set in a small town in Alabama in the 1930s and is narrated by Scout, an outspoken young girl who is the daughter of attorney Atticus Finch. A major conflict is introduced when Atticus defends a black man, Tom Robinson, who has been wrongly accused of assaulting a white woman. As Atticus works to prove Tom's innocence, Scout and her brother Jem experience harassment at the hands of various townspeople.

Mr. Liu is preparing a lesson that includes the following passage from the middle of the novel:

Presently I picked up a comb from Jem's dresser and ran its teeth along the edge.

"Stop that noise," Atticus said.

His curtness stung me. The comb was midway in its journey, and I banged it down. For no reason I felt myself beginning to cry, but I could not stop. This was not my father. My father never thought these thoughts. My father never spoke so. Aunt Alexandra had put him up to this, somehow. Through my tears I saw Jem standing in a similar pool of isolation, his head cocked to one side.

Which of the following is the most likely explanation for why an English-language learner might struggle to comprehend this passage?

- ☐ Confusion over the sequence of events in the passage
- ☐ Difficulty understanding why Scout began to cry for "no reason"
- ☐ Confusion over the sentence "His curtness stung me"
- ☐ Difficulty recognizing that "Atticus" and "my father" refer to the same person

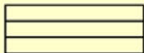
Figure D9 Question #9.

Task	Question Number		Assessment Tools
MET1	10 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

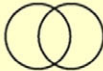
*Questions 10-11 are based on the following teaching scenario.*

Mr. Foreman is teaching his sixth- and seventh-grade students a unit that focuses on strategies for reading informational text. His textbook includes a resource section with ideas that can help students identify the organization of text passages. The following graphic organizers are included in the textbook's resource section:

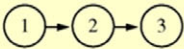
**Description or List**



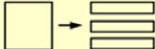
**Compare and Contrast**



**Sequence/Time order**



**Cause and Effect**



Mr. Foreman is considering how to use these examples to help his students understand how graphic organizers can support their comprehension of informational text.

To best help his students improve their reading of informational text, Mr. Foreman should have them use these graphic organizers to

- ☐ record and focus on the meaning of key vocabulary in the text
- ☐ comprehend that nonfiction texts are used to extract factual information
- ☐ identify concrete visual images to recall important details
- ☐ anticipate patterns of information by identifying commonly used structures

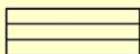
Figure D10 Question #10.

Task	Question Number		Assessment Tools
MET1	11 of 26		<div style="display: flex; justify-content: space-between;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

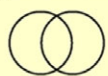
*Questions 10-11 are based on the following teaching scenario.*

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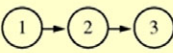
**Description or List**



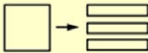
**Compare and Contrast**



**Sequence/Time order**



**Cause and Effect**



Mr. Foreman is considering how to use these examples to help his students understand how graphic organizers can support their comprehension of informational text.

Mr. Foreman is thinking about helpful information to share with his students about these graphic organizers.

For each claim, indicate whether it is an accurate statement.

	Accurate statement	Not an accurate statement
"Cause and Effect" cannot always be used because effects sometimes precede causes in an essay.		
Sometimes more than one graphic organizer can be used because some essays have multiple text structures.		
"Compare and Contrast" is the most useful graphic organizer for nonfiction.		
There are more nonfiction text structures than those presented by these graphic organizers.		
"Description or List" and "Sequence/Time Order" are always chronological.		
These graphic organizers can be used interchangeably.		

Figure D11 Question #11.

Task	Question Number		Assessment Tools
MET1	12 of 26		<div style="display: flex; justify-content: space-between;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

A small group of students in Ms. Varga's advanced ninth-grade English class are reading a book of historical fiction. She wants the students to develop more sophisticated interpretations of the text. To assess their understanding of the text, she gives the following writing assignment.

Choose a character from the novel and write a journal entry that relates the events of the plot from the perspective of your chosen character.

Be sure to:

- accurately reflect the author's portrayal of the character in the novel
- use specific details from the novel to bring your character to life

Explain two ways that this assignment will help Ms. Varga evaluate her students' abilities to interpret the text.

Figure D12 Question #12.



Task	Question Number		Assessment Tools
MET1	13 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 13-17 refer to the following teaching scenario.*

Students in Ms. Joyce's ninth-grade classroom are reading Kurt Vonnegut's short story "Harrison Bergeron" as part of a unit on science fiction. "Harrison Bergeron" is a classic satirical tale written with a heavy dose of irony. In the story, set in 2081, "everyone was finally equal" because the "Handicapper General" has handicapped the more intelligent, athletic, or beautiful members of society. Ms. Joyce is using "Harrison Bergeron" to teach a lesson on irony.

Ms. Joyce anticipates that many of her students will have trouble identifying irony in the story. To help them with this skill, she plans to use the opening paragraph of the story to identify features of Vonnegut's writing that signal to the reader that his story should be read as irony. She reads the opening paragraph aloud:

THE YEAR WAS 2081, and everybody was finally equal. They weren't only equal before God and the law. They were equal every which way. Nobody was smarter than anybody else. Nobody was better looking than anybody else. Nobody was stronger or quicker than anybody else. All this equality was due to the 211th, 212th, and 213th Amendments to the Constitution, and to the unceasing vigilance of agents of the United States Handicapper General.

Ms. Joyce wants to launch an introductory discussion about Vonnegut's use of irony in the paragraph.

Which of the following classroom discussion topics would best help her students focus on Vonnegut's use of irony in the paragraph?

- ☐ Their understanding of constitutional amendments
- ☐ Their ideas about the use of "God and the law" in the passage
- ☐ Their thoughts on why science fiction is a good genre for expressing irony
- ☐ Their perceptions of the meaning of equality in the passage

Figure D13 Question #13. Adapted from *Welcome to the Monkey House* by Kurt Vonnegut, Jr. Copyright © 1961 by Kurt Vonnegut, Jr. Used by permission of Dell Publishing, a division of Random House, Inc.

Task	Question Number		Assessment Tools
MET1	14 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 13-17 refer to the following teaching scenario.*

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Which of the following literary devices should the class focus on as the primary signal that Vonnegut is being ironic in the paragraph?

- ☐ Foreshadowing
- ☐ Hyperbole
- ☐ Repetition
- ☐ Humor

Figure D14 Question #14. Adapted from *Welcome to the Monkey House* by Kurt Vonnegut, Jr. Copyright © 1961 by Kurt Vonnegut, Jr. Used by permission of Dell Publishing, a division of Random House, Inc.

Task	Question Number
MET1	15 of 26

Questions 13-17 refer to the following teaching scenario.

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After reading the paragraph from "Harrison Bergeron," Ms. Joyce asks students to write down what they think the story is going to be about. She stops at one table and looks over the following written responses from three students.

**Akeem writes:** This is going to be a science fiction story in which everybody is better off because everybody is equal. I am not sure how everybody got to be equal in this story. But you can tell that the author thinks it is a good thing because he says everybody was finally equal.

**Lea writes:** This story is going to make fun of the idea that everybody could ever be really equal. It's like when we say everybody is above average. That is just silly. So we know that the author must be kidding.

**Sarah writes:** This is a story about the future. I really like thinking about the future and hoping that everything will be better then. I have never really thought about whether people will be more equal in the future than they are now. I usually just think about pollution and crime and war. Thinking about these things makes me pretty depressed.

Rank the students listed below according to the level of understanding.

To move each student's name to the appropriate box, click on the name, and then click on an empty box. If you change your mind, click on the blue "undo" icon. Scroll down to see the boxes.

	Akeem	Lea	Sarah
Student	Level of understanding		
<div></div>	Best understanding		
<div></div>	Next best understanding		
<div></div>	Most limited understanding		

**Figure D15** Question #15. Adapted from *Welcome to the Monkey House* by Kurt Vonnegut, Jr. Copyright © 1961 by Kurt Vonnegut, Jr. Used by permission of Dell Publishing, a division of Random House, Inc.

Task	Question Number
MET1	16 of 26

Assessment Tool  
Back Review Next

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**Akeem writes:** This is going to be a science fiction story in which everybody is better off because everybody is equal. I am not sure how everybody got to be equal in this story. But you can tell that the author thinks it is a good thing because he says everybody was finally equal.

**Lea writes:** This story is going to make fun of the idea that everybody could ever be really equal. It's like when we say everybody is above average. That is just silly. So we know that the author must be kidding.

**Sarah writes:** This is a story about the future. I really like thinking about the future and hoping that everything will be better then. I have never really thought about whether people will be more equal in the future than they are now. I usually just think about pollution and crime and war. Thinking about these things makes me pretty depressed.

For each student, select one term that best describes his or her response to Vonnegut's paragraph.

	Literal	Interpretive	Personal	Analytic
Akeem				
Lea				
Sarah				

**Figure D16** Question #16. Adapted from *Welcome to the Monkey House* by Kurt Vonnegut, Jr. Copyright © 1961 by Kurt Vonnegut, Jr. Used by permission of Dell Publishing, a division of Random House, Inc.

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
MET1	17 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

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Akeem is a new student in the class. Ms. Joyce knows that he is still mastering English, and she wants to understand his thinking in order to draw him into the class's work. She reviews Akeem's response one more time before considering some possible interpretations.

**Akeem writes:** This is going to be a science fiction story in which everybody is better off because everybody is equal. I am not sure how everybody got to be equal in this story. But you can tell that the author thinks it is a good thing because he says everybody was finally equal.

The most likely interpretation is that Akeem

- ☐ is confusing the narrator's voice with the author's intent
- ☐ does not understand what characterizes different genres
- ☐ does not understand the vocabulary in the paragraph
- ☐ has identified the story as science fiction and reads it through that lens alone

Figure D17 Question #17. Adapted from *Welcome to the Monkey House* by Kurt Vonnegut, Jr. Copyright © 1961 by Kurt Vonnegut, Jr. Used by permission of Dell Publishing, a division of Random House, Inc.

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
MET1	18 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

The ninth-grade students in Ms. Jett's class are writing short stories. The class has been learning to write concisely, including only information that is relevant to the author's purpose. Johanna chooses to write a suspense story. She starts her story with the following description:

I was in a state of complete unawareness and becoming weary. Every tree looked the same, repeating itself over and over again, moving slightly as if a flip book. The path was long gone and now my purpose seemed silly. Earlier the rustle of a squirrel had frightened me but now it was background music and completely insignificant.

The only reason I was there was that Lori—who has everything—had told me she would give me her allowance for five weeks. Lori's allowance is three times as big as mine and her closet is full of pretty dresses with sequins that sparkle like stars. I had my eye on the perfect prom dress but it was too expensive. Lori's allowance would just pay for it, if I ever got back.

Everyone always told me dares were childish and as productive as hitting your head against a wall. My friends had told me I was crazy to be going to Mr. Bloom's woods. Everybody whispered that he was strange, and may have done something bad in the past. Mom wouldn't have put her stamp of approval on this little adventure, but what she didn't know wouldn't hurt her. But I was beginning to wonder if it would hurt me.

Ms. Jett conferences with Johanna to discuss the beginning of her story.

What two suggestions about the second paragraph should Ms. Jett provide to help Johanna improve her story?

Figure D18 Question #18.



Task	Question Number			Assessment Tools
MET1	19 of 26			<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Rice begins a unit on memoir writing by reading a passage from a literary model. She then asks students to complete a warm-up activity to help them generate ideas for their own writing.

For each assignment, indicate whether it will help students focus their brainstorming on generating a memoir.

	Will help focus brainstorming	Will not help focus brainstorming
Write a poem about the ways you have changed, using the form "I used to be...but now I am..."		
Write a sequence of sentences describing some of your experiences, beginning each sentence with the phrase "I remember."		
Write a few adjectives that describe your personality.		
Write down some of your favorite foods and describe what you like about them.		

Figure D19 Question #19.

Task	Question Number			Assessment Tools
MET1	20 of 26			<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Gupta has been working with her eighth-grade students on writing memoirs that focus on a single important event. One student, Jacob, tends to struggle with writing. Ms. Gupta is pleased to see him working enthusiastically on his memoir about going to a NASCAR race. In a one-on-one conference, Ms. Gupta reads Jacob's opening paragraph and recognizes that it represents a significant advance in his writing:

The sky looked like fireworks bursting in a combination of colors: red and blue, green and yellow, pink and green, green and orange, orange and purple, purple and white. The clouds looked like a bunch of exotic cars breezing by my house, with their fully customized bodies. The sun looked like a great big bubble popping in the sky. When the cars started to drive by the pit lane, they were like a group of lazy turtles until the green flag went out. Then they started to move like a group of forty-three road runners.

Which of these follow-up responses from Ms. Gupta would best help Jacob develop a stronger memoir?

- ☐ "I like your vivid use of colors at the beginning. Can you use other details like that to describe your setting?"
- ☐ "I like the way you bring the reader right into the middle of the scene. Can you tell me more about what you were doing while all of this was happening?"
- ☐ "I like how you use all those similes to really describe what you're seeing. Can you supplement them with some metaphors?"
- ☐ "I like the way you show the reader why you think car racing is so exciting. Can you use dialogue to show the crowd's excitement?"

Figure D20 Question #20.

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
<b>MET1</b>	<b>21 of 26</b>		<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">Back</div> <div style="margin-right: 5px;">Review</div> <div>Next</div> </div>

Questions 21 and 22 are based on a writing assignment given to Mr. Johnson's class.

The students in Mr. Johnson's class are writing the first draft of an essay about someone who has influenced them. Mr. Johnson wants each student's essay to have a clear thesis statement and supporting details. He explains, "Your thesis statement should tell why the person has influenced you, and the rest of your essay should show me that whatever you said in your thesis statement is true about that person."

Mr. Johnson circulates around the room and checks students' writing. He looks over the following initial draft from one of his students, named Frank.

**Why My Dad is Cool**  
by Frank

The person who has influenced me is my father because he smart, fun, an nice person. The reason he influenced because he always telling me that you can do it and no one can stop you But you he also let he come and vist him anytime. He support me.

Indicate whether Frank's essay has met each of the following objectives from Mr. Johnson's assignment.

	Has met objective	Has not met objective
Write a clear thesis statement that tells why someone has influenced you.		
Include supporting details about why someone has influenced you.		
Align your thesis statement with supporting details.		

Figure D21 Question #21.

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
<b>MET1</b>	<b>22 of 26</b>		<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">Back</div> <div style="margin-right: 5px;">Review</div> <div>Next</div> </div>

Questions 21 and 22 are based on a writing assignment given to Mr. Johnson's class.

After receiving a few comments from Mr. Johnson, Frank produces the following second draft.

**Why My Dad is Cool**  
by Frank

The person who has influenced me is my father because he smart, fun, an nice person. The reason he influences because he always telling me that you can do it and no one can stop you But you he also let me come and vist him anytime I get ready Another reason is because he The way he dress is like they do in the army he is 6 8 weigh about 195 and a basketball player. One day me and him play one on one he use to play ball. He tell me that you can become the best player then any body One day he show me how to play the game and how to post up to other players and dunk. He went to school at Broad Street High School were he won the first gold ball for Broad Street he was a strong A's student from High School to collage were he won the gold ball at FCULA collage in California were he won another gold ball 1987.

Mr. Johnson realizes that Frank is having trouble with this assignment. He wants to select a focus for Frank's next revision to help him improve the quality of his essay.

Which of the following would be the most important for Frank to address in his next revision?

☐ Spelling and punctuation

☐ Grammar and verb tense

☐ Staying on topic

☐ Organization of ideas

Figure D22 Question #22.

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
MET1	23 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Mr. Jackson's sixth- and seventh-grade students are writing flash fiction. After reading the following draft of Arielle's short story, Mr. Jackson wants to point out some of the strengths in her writing.

Natasha left the house and walked sadly down the sidewalk, slowly shuffling her feet. As she walked by the old house, she was surprised by how sad she felt. She stepped onto the grass, and the sprinklers sprayed her face. The sprinkle of cold water caught her by surprise. Her brother shouted down the street asking her to wait up. Thankfully he couldn't tell the tears from the drops of water.

For each writing feature, indicate whether it describes a strength in Arielle's writing.

	Describes a strength in Arielle's writing	Does not describe a strength in Arielle's writing
Strong character development		
Use of figurative language		
Conveying emotions through showing		
Use of powerful images		

Figure D23 Question #23.

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
MET1	24 of 26		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

*Questions 24-25 refer to the following teaching scenario.*

Mr. Perez is preparing a lesson that requires his seventh-grade students to read an informational text in their English anthology about the sinking of the Titanic. The text is an excerpt from *Exploring the Titanic* by Robert Ballard, an example of literary nonfiction that uses an informational narrative format. Instructional activities in the anthology focus on distinguishing between fact and opinion. Although Mr. Perez thinks this is a useful skill, he wants his students to think more deeply about the craft of writing informational text. He brainstorms a list of possible activities to help students increase their knowledge of narrative features of nonfiction text.

For each student activity, indicate whether it is likely to help Mr. Perez's students increase their knowledge of narrative features of this informational text.

	Likely to help	Not likely to help
Describing how the author uses sequential text structure to tell the story of the Titanic		
Identifying and tracking unfamiliar naval vocabulary words and trying to determine their meaning from context clues		
Analyzing how the author uses a description of the Titanic to reflect the social values of the early 1900s		
Explaining why the author chooses eyewitness accounts to describe what happened during the sinking of the Titanic		

Figure D24 Question #24.



Task	Question Number		Assessment Tools
MET1	25 of 26	<p><i>Questions 24-25 refer to the following teaching scenario.</i></p> <p>Mr. Perez is preparing a lesson that requires his seventh-grade students to read an informational text in their English anthology about the sinking of the Titanic. The text is an excerpt from <i>Exploring the Titanic</i> by Robert Ballard, an example of literary nonfiction that uses an informational narrative format. Instructional activities in the anthology focus on distinguishing between fact and opinion. Although Mr. Perez thinks this is a useful skill, he wants his students to think more deeply about the craft of writing informational text. He brainstorms a list of possible activities to help students increase their knowledge of narrative features of nonfiction text.</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Mr. Perez asks his students to analyze the following paragraph from the book:</p> <p>For the next ten months the Titanic was outfitted and carefully prepared down to the last detail. The final size and richness of this new ship was astounding. She was 882 feet long, almost the length of four city blocks. With nine decks, she was as high as an eleven-story building.</p> </div> <p>In keeping with his goal of focusing students' attention on the craft of writing informational text, which of the following questions should Mr. Perez ask the class?</p> <ul style="list-style-type: none"> <li><input type="radio"/> "How does the author help us understand the size of the Titanic?"</li> <li><input type="radio"/> "How long do you think it would take to build the Titanic today?"</li> <li><input type="radio"/> "How do the details in this paragraph help the author emphasize the difficulties in building the Titanic?"</li> <li><input type="radio"/> "How would you describe the size of the Titanic?"</li> </ul>

Figure D25 Question #25.

Task	Question Number		Assessment Tools
MET1	26 of 26	<p>Ms. Lee is teaching a unit on persuasion to her sixth- and seventh-grade students. Although the students have read articles on the pros and cons of their topic—eliminating vending machines from school cafeterias—she notices that they have not used textual evidence in their essays.</p> <p>Which of the following is the most effective activity to help Ms. Lee's students add textual evidence to their essays to support their arguments?</p> <p>Ms. Lee should ask her students to</p> <ul style="list-style-type: none"> <li><input type="radio"/> critically read articles about the pros and cons of having vending machines in schools</li> <li><input type="radio"/> record evidence they find in articles on sticky notes and place the sticky notes in the appropriate places in their drafts</li> <li><input type="radio"/> peer edit each others' drafts with a focus on identifying both strengths and weaknesses in the essays</li> <li><input type="radio"/> use their writing guides to apply the proper citation style for authoritative sources</li> </ul>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Back Review Next</p> </div>

Figure D26 Question #26.

## Appendix E

### Algebra I Assessment

Table E1 Item/Question Sequence Number for Algebra I Assessment

Item/question sequence number	Answer key	Item/question sequence number	Answer key
1	B	11d	Valid
2	C	11e	Not valid
3a	Provides	12	B
3b	Does not provide	13	B
3c	Provides	14	C
3d	Does not provide	15a	Provides
3e	Provides	15b	Provides
4	C	15c	Provides
5	C	15d	Does not provide
6a	Would not support	15e	Does not provide
6b	Would support	16	C
6c	Would support	17	C
7	B	18	B
8	C	19	D
9	C	20	C
10	B	21	D
11a	Valid	22	D
11b	Valid	23	C
11c	Not valid		

<b>Task</b>	<b>Question Number</b>		<b>Assessment Tools</b>
MET1	1 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Mr. Wright asked his students to solve the equation  $6 - 3(x - 5) = 24$ . After reviewing his students' work, he found two interesting methods for solving the equation and asked those two students to present their methods on the board.

<p><u>Brenda's method</u></p> $  \begin{aligned}  6 - 3(x - 5) &= 24 \\  6 - 3x + 15 &= 24 \\  -3x + 21 &= 24 \\  -3x &= 3 \\  x &= -1  \end{aligned}  $	<p><u>Daniel's method</u></p> $  \begin{aligned}  6 - 3(x - 5) &= 24 \\  -3(x - 5) &= 18 \\  x - 5 &= -6 \\  x &= -1  \end{aligned}  $
--	--

After Brenda and Daniel presented their methods, Mr. Wright's class discussed these two methods. One student, Steve, compared them and then said, "I like Daniel's method because there are less steps. However, if it is a harder division, Brenda's method would be easier." For which of the following equations would Steve be most likely to use Brenda's method?

☐  $5 - 2(x - 3) = 21$   
☐  $10 - 7(x + 5) = 6$   
☐  $6 - 3(x + 1) = 9$   
☐  $8 + 4(x - 3) = 14$

Figure E1 Question #1.

Task	Question Number	Assessment Tools
MET1	2 of 23	<a href="#">Back</a> <a href="#">Review</a> <a href="#">Next</a>

A lesson in Ms. Hagerman's textbook defines the distributive property, but the exercises merely ask for its definition. To motivate her students to learn the definition, Ms. Hagerman tells them that the distributive property can often be used to simplify the evaluation of expressions.

She wants to give her students an example that will focus their attention on how the distributive property can be useful in evaluating expressions. Of the following expressions, which would best serve her purpose?

☐  $12 \times \left( \frac{3}{4} + \frac{1}{4} \right)$

☐  $18 \times \left( \frac{3}{5} - \frac{1}{10} \right)$

☐  $36 \times \left( \frac{5}{12} + \frac{2}{9} \right)$

☐ Each of these expressions would serve her purpose equally well.

Figure E2 Question #2.

Task	Question Number	Assessment Tools
MET1	3 of 23	Back Review Next

During a lesson on solving multistep equations, Ms. Kane asked her students to solve the equation  $-5x + 8 = 13x - 10$ . While walking around the classroom looking at what the students were writing, she noticed several different strategies. For each of the following student solutions, indicate whether or not the work provides evidence that the student is reasoning correctly about this problem.

	Provides Evidence of Correct Student Reasoning	Does Not Provide Evidence of Correct Student Reasoning
$  \begin{aligned}  -5x + 8 &= 13x - 10 \\  8 &= 18x - 10 \\  18 &= 18x \\  1 &= x  \end{aligned}  $		
$  \begin{aligned}  -5x + 8 &= 13x - 10 \\  \frac{3x}{3} &= \frac{3}{3} \\  x &= 1  \end{aligned}  $		
$  \begin{aligned}  -5x + 8 - 13x + 10 &= 13x - 10 - 13x + 10 \\  -5x - 13x + 8 + 10 &= 0 \\  -18x + 18 - 18 &= 0 - 18 \\  \frac{-18x}{-18} &= \frac{-18}{-18} \cdot \frac{x}{-18} \\  x &= 1  \end{aligned}  $		
$  \begin{aligned}  -5x + 8 &= 13x - 10 \\  -5x + 8 &= 3x \\  +8x &+ 5x \\  \frac{8}{8} &= \frac{8x}{8} \\  1 &= x  \end{aligned}  $		
$  \begin{aligned}  -5x + 8 &= 13x - 10 \\  -13x + 8 &= 13x - 8 \\  \frac{-18x}{-18} &= \frac{-18}{-18} \\  x &= 1  \end{aligned}  $		

Figure E3 Question #3.

Task	Question Number		Assessment Tools
MET1	4 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Mr. Brownstein's class was solving the equation:

$$3(x - 2)^2 = 6(x - 2)(x + 5).$$

Kenneth suggested dividing both sides by 3 to get:

$$(x - 2)^2 = 2(x - 2)(x + 5).$$

Then he suggested dividing both sides by  $(x - 2)$ , but Sandra said, "You cannot divide both sides by  $(x - 2)$ ." In response, Kenneth asked, "If you can divide both sides by 3, why can't you divide both sides by  $(x - 2)$ ?"

Of the following statements, which best explains why you cannot divide both sides of the equation by  $(x - 2)$  as Kenneth suggested?

- ☐ You cannot cancel  $(x - 2)$  because it represents a real number.
- ☐ It is better to expand the expressions on both sides of the equation first to obtain  $x^2 - 4x + 4 = 2(x^2 + 3x - 10)$ , and then you won't have to worry about  $(x - 2)$ .
- ☐ Division by zero is not defined, so you would have to consider the case of  $x = 2$  separately.
- ☐ Because  $x$  is a variable, it can vary—you may not be canceling the same amount from both sides.

Figure E4 Question #4.

Task	Question Number		Assessment Tools
MET1	5 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

During a unit on solving linear equations, Ms. Martino asks her students to write one question that she could use on the unit test at the end of the chapter. Joe writes the following question.

If Joe had to solve the sentence  $8y - 9 = 0$  for  $y$ , what would be the value of  $y$  ?

Ms. Martino thinks the equation would be a good one for her students to solve. However, she decides to revise the question because she is concerned that the wording may cause some of her English Language Learner (ELL) students to answer this question incorrectly, even if they understand the mathematics involved. Of the following revisions, which one best addresses Ms. Martino's concern?

- ☐ When Joe solved for  $y$  in the equation  $8y - 9 = 0$ , what was the value of  $y$  ?
- ☐ Joe solved the sentence  $8y - 9 = 0$  for  $y$ . What is the value of  $y$  ?
- ☐ What is the value of  $y$  in the equation  $8y - 9 = 0$  ?
- ☐ If  $8y - 9 = 0$ , what would be the value of  $y$  ?

Figure E5 Question #5.

Task	Question Number	Assessment Tools	
MET1	6 of 23	Back	Review Next

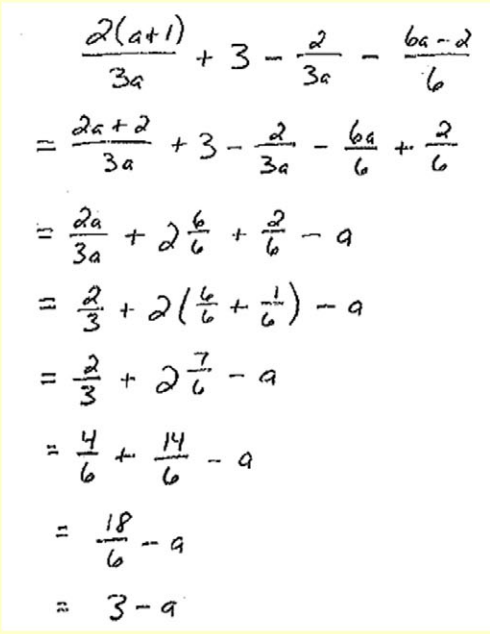
Before teaching a lesson on multiplying two trinomials, Ms. Ryan wants a better sense of what her students know about multiplying two binomials. She asks them to find the product  $(2x + 1)(x - 4)$  and explain their methods. While walking around the class, she notices several different methods. For each of the following, indicate whether or not the student response provides evidence that the student has an understanding of the multiplication of binomials that would support the development of a strategy for multiplying two trinomials.

	Would Support a Strategy for Trinomials	Would Not Support a Strategy for Trinomials
<p>I used the FOIL method. The product of the first terms is <math>2x^2</math>, the product of the outer terms is <math>-8x</math>, the product of the inner terms is <math>x</math>, and the product of the last terms is <math>-4</math>. Then, I combined the two like terms, the <math>-8x</math> and the <math>x</math>, to get <math>-7x</math>, so my final answer is <math>2x^2 - 7x - 4</math>.</p> <div style="text-align: center;"> </div>		
<p>I just multiplied like you do with regular numbers. I put the <math>2x + 1</math> on the top and the <math>x - 4</math> on the bottom. Then I multiplied each term on the bottom by each one on the top. Finally, I added up the like terms to get my answer, <math>2x^2 - 7x - 4</math>.</p> <div style="text-align: center;"> </div>		
<p>I drew a box with two columns for the <math>2x</math> and the <math>1</math> and two rows for the <math>x</math> and the <math>-4</math>. Then, I multiplied the terms that went with the row and column that each small box was in. Then, I wrote out what I got in each small box, combined the like terms, and got <math>2x^2 - 7x - 4</math>.</p> <div style="text-align: center;"> </div>		

Figure E6 Question #6.



Task	Question Number		Assessment Tools
MET1	7 of 23		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>



Mr. Anderson asked his students to simplify the following algebraic expression.

$$\frac{2(a+1)}{3a} + 3 - \frac{2}{3a} - \frac{6a-2}{6}$$

One of his students gave the incorrect solution shown to the left.

Of the following descriptions, which best characterizes what is wrong with this student's work?

- ☐ This student used the distributive property incorrectly.
- ☐ This student confounded mixed fractions with factors.
- ☐ This student forgot to cancel common factors in several places.
- ☐ This student needs to apply a more formal procedure by finding the common denominator and then adding all terms.

Figure E7 Question #7.

Task	Question Number		Assessment Tools
MET1	8 of 23		<input type="button" value="Back"/> <input type="button" value="Review"/> <input type="button" value="Next"/>

Having nearly finished a chapter on linear equations, Mr. Hassan's students seem quite proficient in generating standard formats for linear equations, using techniques for graphing linear equations, and solving systems of two linear equations. However, he is concerned that his students are applying these techniques in routine ways and tend to think only algebraically or only geometrically without reasoning fluidly in both ways.

Mr. Hassan wants to give his students a problem that would require an understanding of the topic that goes beyond the set of procedures students have learned and that would support his students' ability to work and talk across algebraic and geometric interpretations. Of the following problems, which would best serve this dual purpose?

- ☐ Describe in your own words a procedure for finding the point of intersection given the equations of two lines.
- ☐ Find the intersection of the following two lines and graph them.
 
$$y = 2x + 3$$

$$y = 2x - 7$$
- ☐ Consider two linear functions, where  $a$  and  $b$  are negative.
 
$$y = x + 3$$

$$y = ax + b$$

What can you say about the point of intersection of their graphs?
- ☐ Using ideas about solving systems of two linear equations, solve the following system of three equations and explain what the solution means.
 
$$x + 4y + z = 0$$

$$x - 4y + 2z = 3$$

$$x = 4y + z$$

Figure E8 Question #8.

Task	Question Number		Assessment Tools
MET1	9 of 23	<p>Having taught her students to factor quadratics with integer coefficients, integer roots, and a leading coefficient of 1, Ms. Quezada explained that she was going to give them a harder problem. She then asked them to solve the following.</p> $3x^2 - 3x - 6 = 0$ <p>After a few minutes of work, the class discussed their solutions. Letitia said that <math>x</math> was <math>-1</math> or <math>2</math> and explained, "I added <math>3x</math> to both sides and divided by <math>3</math>."</p> $3x^2 - 6 = 3x$ $x^2 - 2 = x$ <p>She then continued, "The parabola's just down a little and the line's at 45 degrees, so it's just below zero and about 2 to the right. <math>x</math> can be <math>-1</math> and <math>2</math>, and those are the only possible ones."</p> <p>Of the following, which best characterizes Letitia's approach to this problem?</p> <ul style="list-style-type: none"> <li><input type="radio"/> Letitia's <u>method</u> is wrong because she should have first divided by 3 and then factored the left side of the equation.</li> <li><input type="radio"/> Letitia's <u>method</u> is wrong because this is a parabola and you could graph it, but you would have to graph the original equation and look for the roots.</li> <li><input type="radio"/> Letitia's <u>reasoning</u> is correct, but her method often leads to points of intersection that might be hard to determine visually.</li> <li><input type="radio"/> Letitia's <u>reasoning</u> is correct, but her method requires knowledge of calculus.</li> </ul>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">Back</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin: 0 5px;">Review</div> <div style="border: 1px solid black; padding: 2px; display: inline-block;">Next</div>

Figure E9 Question #9.

Task	Question Number		Assessment Tools
MET1	10 of 23	<p>Ms. Lang's class had been studying the concept of slopes of lines, so she asked them to consider all of the lines passing through one point and how the slopes of those lines vary. The students had used geoboards in some earlier work, so they started talking about the slopes of lines on an "infinitely extended" geoboard. (Geoboards are flat blocks of wood, roughly one foot square, with pegs laid out on a grid where rubber bands can be hooked to make lines or polygons.) The students decided that the pegs of the infinite geoboard could be thought of as the set of points with integer coordinates in the Cartesian plane.</p> <p>During the discussion, students had the following exchange.</p> <p>Yonah: On the geoboard, you can't get all of the slopes, because the geoboard points are too spread out—there are a whole bunch of lines between the ones you can make.</p> <p>Andy: I disagree. I think we can make any slope. Starting at one point, by choosing another geoboard point far enough away, we can tilt the line as much or as little as we like.</p> <p>Becky: What I was thinking was if you run a line through one geoboard point, it will always hit another one far enough out.</p> <p>Of the following concepts, which is most directly related to the mathematics underlying this discussion?</p> <ul style="list-style-type: none"> <li><input type="radio"/> Interpretation of the derivative—the derivative is the slope of the tangent line.</li> <li><input type="radio"/> Density of numbers on the real line—the rational numbers are dense, but not every real number is rational.</li> <li><input type="radio"/> The parallel postulate—given a point and a line, there is a unique line through the given point parallel to the given line.</li> <li><input type="radio"/> Each of these concepts is equally related to the underlying mathematics.</li> </ul>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">Back</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin: 0 5px;">Review</div> <div style="border: 1px solid black; padding: 2px; display: inline-block;">Next</div>

Figure E10 Question #10.

Task	Question Number	Assessment Tools
MET1	11 of 23	Back Review Next

During a lesson on solving multistep equations, Mr. Steinbrecher asked his students to solve the equation  $4(5x - 11) = 16$ . While walking around the class looking at what the students were writing, he noticed several different strategies. For each of the following student solutions, indicate whether or not it is a valid strategy for solving this problem.

	Strategy Is Valid	Strategy Is Not Valid
$\frac{1}{4} \cdot 4(5x - 11) = 16 \cdot \frac{1}{4}$ $5x - 11 = 4$ $5x = 15$ $x = 3$		
$4 \cdot 4(5x - 11) = 16 \cdot 4$ $\frac{16(5x - 11)}{16} = \frac{16 \cdot 4}{16}$ $5x - 11 = 4$ $\frac{+11}{+11}$ $5x = 15$ $x = 3$		
$4(5x - 11) = 16$ $9x - 11 = 16$ $\frac{+11}{+11}$ $\frac{9x}{9} = \frac{27}{9}$ $x = 3$		
$4(5x - 11) = 16$ $\frac{20x - 44}{20} = \frac{16}{20}$ $+ \frac{44}{20} + \frac{44}{20}$ $x = \frac{60}{20}$ $x = 3$		
$\frac{4}{H} \left( \frac{5x}{4} - \frac{11}{4} \right) = \frac{16}{4}$ $\frac{5x}{4} - \frac{11}{4} = 4$ $+ \frac{11}{4} + \frac{11}{4}$ $\frac{H}{8} \cdot \frac{5x}{4} = \frac{18}{4} \cdot \frac{H}{8}$ $x = 3$		

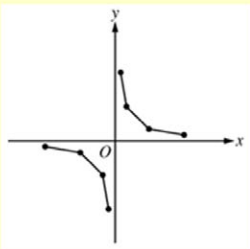
Figure E11 Question #11.

Task	Question Number		Assessment Tools
MET1	12 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Mr. Jakobsen's students were graphing the function below, where  $y$  is inversely proportional to  $x$ .

$$y = \frac{1}{x}$$

One of his students drew the following graph.



Mr. Jakobsen has noticed that students often draw graphs with line segments like this despite frequent reminders that the graph should be curved. To get his students to discuss this issue, he asked the class what was wrong with the drawing. Of the following student explanations, which provides the best mathematical explanation of why drawing connected line segments is inappropriate for this graph?

- ☐ When the  $x$  changes, the graph should change at the same rate all the time and it shouldn't have corners.
- ☐ The graph changes all the time, but it cannot have sudden changes at some of the points.
- ☐ For any whole number,  $\frac{1}{x}$  will always be a rational number and that makes it hard to draw the graph for irrational numbers.
- ☐ The problem is that you don't have enough points. You need to include more points to make it look correct.

Figure E12 Question #12.

Task	Question Number		Assessment Tools
MET1	13 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

In a unit on simplifying expressions, one of Mr. Serrano's students wrote the following correct solution.

$$\begin{aligned}
 & \frac{4(a+2)}{3a} + 2 - \frac{8}{3a} - \frac{6a-1}{6} \\
 &= \frac{4a+8}{3a} + 2 - \frac{8}{3a} - \frac{6a}{6} + \frac{1}{6} \\
 &= \frac{4a}{3a} + \frac{1}{6} - a + \frac{1}{6} \\
 &= \frac{1}{3} + \frac{1}{6} - a \\
 &= \frac{1}{2} + \frac{1}{6} - a \\
 &= \frac{2}{3} - a \\
 &= \frac{3}{6} - a \\
 &= \frac{1}{2} - a
 \end{aligned}$$

Of the following descriptions, which best characterizes the student's work?

- ☐ The student knows how to simplify expressions very well and demonstrates strategic use of standard procedures.
- ☐ The student shows good computational skill but does not use processes efficiently.
- ☐ The student knows how to simplify expressions very well, but in the solution the student should write all steps involved in the calculation, such as the step  $\frac{8}{3a} - \frac{8}{3a}$ .
- ☐ The student should apply a more formal procedure by first finding the common denominator and then adding all terms.

Figure E13 Question #13.

Task	Question Number		Assessment Tools
MET1	14 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

A lesson in Ms. Taylor's textbook states the associative and commutative properties of addition. To motivate the students to learn the properties, she tells her students that the properties can often be used to simplify the evaluation of expressions.

She wants to give her students an example that will focus their attention on how these properties can be useful in evaluating expressions. Of the following expressions, which would best serve her purpose?

☐  $(455 + 456) + (457 + 458)$

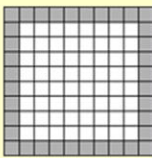
☐  $(647 + 373) + (227 + 456)$

☐  $(551 + 775) + (49 + 225)$

☐ Each of these expressions would serve her purpose equally well.

Figure E14 Question #14.

Task	Question Number		Assessment Tools
MET1	15 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>



Ms. Kamp asks her students to consider squares of different side lengths with only the boxes along the sides shaded as in the figure on the left.

She asks each student to write an expression for the number of shaded boxes in a square with a side length of  $n$  boxes and to explain why the expression gives the number of shaded boxes for any size square. For each of the following explanations, indicate whether or not it provides evidence that the student understands why the expression can be used to find the number of such shaded boxes in any square.

	Provides Evidence	Does Not Provide Evidence
If you start on the bottom left and go to just below the top left, then start with the top left and go to just before the top right, and keep doing that, you will get 4 groups, and each group has 1 less than $n$ , so you get $4(n - 1)$ .		
My expression is $n + 2(n - 1) + (n - 2)$ because the top of the square has $n$ shaded boxes, then each of the sides has $n - 1$ shaded boxes left, and then the bottom has $n - 2$ shaded boxes left.		
Inside the square with a side length of $n$ boxes is a square with side length of $n - 2$ boxes, so if you find the area of the two squares and subtract them, you will find the number of shaded boxes. So, I get $n^2 - (n - 2)^2$ .		
I get $4(n - 2) + 4$ because there are 36 boxes shaded, and when you put 10 in for the $n$ in $4(n - 2) + 4$ and follow the order of operations, the answer is 36.		
My expression is $2n + 2(n - 2)$ because if you simplify $2n + 2(n - 2)$ you get $2n + 2n - 4$ , which is equal to $4n - 4$ , and because this doesn't depend on $n$ , it works for any $n$ .		

Figure E15 Question #15.



Task	Question Number		Assessment Tools
MET1	16 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Quinn asked her students to solve the following quadratic equation.

$$3x^2 - 6x - 24 = 0$$

Maurice explained, "I added 24 to both sides and divided by 3."

$$3x^2 - 6x = 24$$

$$x^2 - 2x = 8$$

$$x(x - 2) = 8$$

He then concluded, "The only numbers that are 2 apart and multiply to be 8 are 2 and 4, and -2 and -4, so  $x$  has to be 4 or -2." Students agreed that 4 and -2 work when you substitute them into the original equation, but they were unsure about his method.

Of the following statements, which best characterizes Maurice's approach to this problem?

- ☐ Maurice's method is wrong because you cannot solve an equation by factoring unless one side of the equation is equal to zero.
- ☐ Maurice's method is wrong because he should have first divided by 3 and then factored the left side of the equation.
- ☐ Maurice's reasoning is correct, but his method often leads to an equation that cannot be solved by inspection.
- ☐ Maurice's reasoning is correct, but his method only works for equations with real roots.

Figure E16 Question #16.

Task	Question Number		Assessment Tools
MET1	17 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

In the last class, Mr. Rosen's students graphed quadratics of the form  $y = x^2 + c$  for various values of  $c$  and developed a rule about shifting the graph of  $y = x^2$  up or down. Today Mr. Rosen asked the students to predict what would happen when they graphed  $y = (x - 3)^2$  and then to graph it. Students were surprised that the graph shifted 3 units to the right rather than left or down as they had predicted. Mr. Rosen asked them to explore a little further in groups.

As he walked around the classroom, each group explained to him the strategy they were using to explore the problem. Of the following student descriptions of a strategy for exploring the problem, which is most directly related to the underlying mathematical reason for the graph's behavior?

- ☐ We are trying to prove the rule, so each of us is graphing another one,  $y = (x - 2)^2$ ,  $y = (x - 5)^2$ ,  $y = (x + 2)^2$ , and  $y = (x + 1)^2$ , and then we will compare our results.
- ☐ We are making a table like yesterday and putting  $x$  and  $x^2$  and  $(x - 3)^2$ , so we can plug in different inputs and compare what the outputs are in  $x^2$  and  $(x - 3)^2$ .

$x$	$x^2$	$(x - 3)^2$
0		
1		

- ☐ We decided to look at the roots of  $y = x^2$  and  $y = (x - 3)^2$ . The vertex of  $y = (x - 3)^2$  is  $(3, 0)$ , and the vertex of  $y = x^2$  is  $(0, 0)$ . We are looking to see what  $x$ -values we have to put in  $(x - 3)^2$  to make the outputs the same as in  $x^2$ .
- ☐ We are going to FOIL it out so it will look more like the ones from yesterday, and then we can graph it and compare the way we did yesterday.

Figure E17 Question #17.



Task	Question Number		Assessment Tools
MET1	18 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Seidel is introducing the distributive property. To motivate her students, she wants to give them an example that will focus their attention on how using the distributive property can simplify computations. In which of the following examples will the use of the distributive property most simplify the computations?

☐  $12 \times 29 + 12 \times 38 = \underline{\hspace{2cm}}$   
☐  $17 \times 37 + 17 \times 63 = \underline{\hspace{2cm}}$   
☐  $13 \times 13 + 15 \times 15 = \underline{\hspace{2cm}}$   
☐  $16 \times 24 + 16 \times 24 = \underline{\hspace{2cm}}$

Figure E18 Question #18.

Task	Question Number		Assessment Tools
MET1	19 of 23		<div style="display: flex; justify-content: space-around;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Christensen asked Carla to simplify the following expression.

$$\frac{10a + 4}{2a}$$

Carla wrote the following incorrect solution.

$$\frac{10a + 4^2}{2a} = 10 + 2 = 12$$

Ms. Christensen then asked the class what was wrong with this solution. Which of the following student explanations characterizes what was most likely wrong with Carla's solution?

☐ She should have written  $2(5a + 2)$  in the top and then canceled the 2s in the top and the bottom.  
☐ She saw you can break the fraction into two fractions, but the way she simplified each fraction is wrong.  
☐ She divided the 4 by 2, but you cannot cancel when you have more than one thing added in the top.  
☐ It's not possible to cancel like this because you can only cancel factors that are the same for the top and the bottom.

Figure E19 Question #19.

Task	Question Number		Assessment Tools
MET1	20 of 23		<div style="display: flex; justify-content: space-between;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Ms. Lindsey's textbook uses a geoboard to model slope as rise over run. (Geoboards are flat blocks of wood, roughly one foot square, with pegs laid out on a grid where rubber bands can be hooked to make lines or polygons.) As her students explored different slopes they could make on the geoboard, Edward asked, "Since the diagonal of one of the unit squares has length  $\sqrt{2}$ , does that mean you can make a line segment with slope  $\sqrt{2}$  on the geoboard?" When Ms. Lindsey asked the class whether they thought this could be done, the following exchange occurred.

Andy: Edward's right that the diagonal of the unit square has length  $\sqrt{2}$ , but its slope is 1.

Beth: Well, that doesn't matter. We can just turn the geoboard so that the diagonal is horizontal, and then we can see squares with side length  $\sqrt{2}$ .

Caitlin: Sure, but the square roots of two would just cancel. I think they always would, so you can't get  $\sqrt{2}$  as a slope.

Dan: That's not right, because we can make one length of  $\sqrt{2}$  and another length of 1 and use them as the rise and the run.

Which of the student statements gives the best insight into Edward's question?

☐ Andy's statement

☐ Beth's statement

☐ Caitlin's statement

☐ Dan's statement

Figure E20 Question #20.

Task	Question Number		Assessment Tools
MET1	21 of 23		<div style="display: flex; justify-content: space-between;"> <span>Back</span> <span>Review</span> <span>Next</span> </div>

Mr. Roberts' class has been learning to factor quadratics. His students know that the polynomial  $x^2 - 4x + 3$  can be factored into  $(x - 3)(x - 1)$ , and he wants to build on this knowledge to factor special cases of higher-degree polynomials that are quadratic in form. He starts off with  $x^4 - 4x^2 + 3 = ( ) ( )$  and asks his students to suggest powers of  $x$  that can be put into each set of parentheses.

Carmen suggests that the first term in the first set of parentheses could be  $x^3$ , and the first term in the second set could be  $x$ . Of the following statements, which best characterizes Carmen's proposal?

☐ Her idea for factoring the quartic will not work, because no matter how you complete the factors, when you multiply it out, you'll end up with an  $x^3$ .

☐ Her idea for factoring the quartic will not work, because the assumption in this context is that the polynomials are over the real numbers, not over the complex numbers.

☐ Her idea for factoring the quartic will work, but you have to choose the other two numbers correctly. If you use  $-3$  and  $-1$ , when you multiply it out, you get  $x^4$  and  $-1$  times  $-3$  equals 3.

☐ Her idea for factoring the quartic will work, but it produces a factorization that is not of the same form as the quadratics that the class has already learned to factor.

Figure E21 Question #21.

Task	Question Number		Assessment Tools
MET1	22 of 23	<p>Ms. Collingwood is teaching a unit on graphing. Some of the students in her class speak a language other than English at home and when they work in small groups. As she prepares for the unit, she makes a note of vocabulary words that she believes will be challenging for her students, especially words that have different meanings in different contexts. Of the following vocabulary words that she identified, which will require the <u>least</u> clarification regarding differences in meaning?</p> <p> <input type="radio"/> Plane  <input type="radio"/> Origin  <input type="radio"/> Intercept  <input type="radio"/> Parabola         </p>	<div style="border: 1px solid black; padding: 2px; display: inline-block;"> <div style="display: flex; justify-content: space-between; align-items: center;"> <span>Back</span> <span>Review</span> <span>Next</span> </div> </div>

Figure E22 Question #22.

Task	Question Number		Assessment Tools
MET1	23 of 23	<p>Mr. Baas' students were solving inequalities. Cheryl wrote the following solution on the board.</p> $\frac{x-2}{x} < 1$ $x-2 < x$ $0 < 2$ <p>She concluded that because this is always true, every <math>x</math> would work. After some discussion, students decided that Cheryl's solution was not correct, but they were unsure why. Of the following explanations, which best identifies what is problematic about Cheryl's work on the problem?</p> <p> <input type="radio"/> It's true that 2 is always greater than 0, but because you have eliminated all <math>x</math>, we cannot say what <math>x</math> is.  <input type="radio"/> We can see that the numerator always is two less than the denominator, so the fraction will always be less than 1 for all <math>x</math>. However, we have to require the denominator <math>x \neq 0</math>.  <input type="radio"/> You don't know what <math>x</math> is, so when you multiply with <math>x</math> like this, you must assume <math>x &gt; 0</math>.  <input type="radio"/> She should have simplified the left-hand side of the inequality to <math>1 - \frac{2}{x}</math> and then subtracted 1 from both sides and added <math>\frac{2}{x}</math> to both sides. This would yield <math>0 &lt; \frac{2}{x}</math>, which is true for <math>x &gt; 0</math>.         </p>	<div style="border: 1px solid black; padding: 2px; display: inline-block;"> <div style="display: flex; justify-content: space-between; align-items: center;"> <span>Back</span> <span>Review</span> <span>Next</span> </div> </div>

Figure E23 Question #23.

## Appendix F

## Item Statistics for Content Knowledge for Teaching (CKT) Assessments

Table F1 Mathematics 4–5 Item Statistics

Item/question sequence number	P-plus value	Biserial correlation
1	0.73	0.49
2	0.56	0.51
3a	0.78	0.49
3b	0.65	0.40
3c	0.73	0.51
3d	0.67	0.30
3e	0.59	0.48
4	0.39	0.58
5	0.71	0.46
6	0.54	0.57
7a	0.73	0.19
7b	0.81	0.44
7c	0.76	0.52
7d	0.52	0.25
7e	0.86	0.43
8	0.29	0.47
9	0.61	0.39
10a	0.96	0.47
10b	0.82	0.31
10c	0.50	0.46
10d	0.47	0.36
10e	0.63	0.11
11	0.42	0.36
12	0.19	0.47
13	0.33	0.20
14a	0.46	0.44
14b	0.81	0.52
14c	0.43	0.39
14d	0.83	0.42
14e	0.57	0.59
15	0.26	0.50
16	0.23	0.46
17	0.12	0.56
18	0.13	0.17
19a	0.74	0.42
19b	0.56	0.16
19c	0.56	0.34
19d	0.28	0.06
20	Constructed-response item	
21	Constructed-response item	

Table F2 Mathematics 6–8 Item Statistics

Item/question sequence number	P-plus value	Biserial correlation
1	0.58	0.68
2	0.62	0.42
3a	0.98	0.35
3b	0.71	0.53
3c	0.82	0.59
4	0.33	0.63
5	0.64	0.31
6a	0.84	0.36
6b	0.86	0.62
6c	0.92	0.70
6d	0.69	0.45

Table F2 Continued

Item/question sequence number	P-plus value	Biserial correlation
6e	0.91	0.47
7	0.38	0.38
8a	0.93	0.67
8b	0.66	0.58
8c	0.95	0.56
8d	0.73	0.37
8e	0.79	0.41
9	0.57	0.44
10	0.29	0.37
11a	0.76	0.58
11b	0.84	0.54
11c	0.79	0.61
11d	0.62	0.47
11e	0.86	0.45
11f	0.75	0.35
12	0.39	0.51
13	0.68	0.57
14a	0.53	0.51
14b	0.46	0.57
14c	0.35	0.28
14d	0.61	0.32
15	0.27	0.49
16a	0.74	0.41
16b	0.72	0.41
16c	0.65	0.49
16d	0.56	0.55
16e	0.43	0.32
17	0.38	0.22
18	0.21	0.43
19a	0.42	0.04
19b	0.95	0.50
19c	0.86	0.50
19d	0.61	0.55
20	0.18	0.34
21	0.09	0.12
22	Constructed-response item	
23	Constructed-response item	

Table F3 Algebra I Item Statistics

Item/question sequence number	P-plus value	Biserial correlation
1	0.76	0.53
2	0.55	0.68
3a	0.98	0.02
3b	1.00	— <sup>a</sup>
3c	0.80	0.58
3d	0.88	0.61
3e	0.99	0.71
4	0.77	0.63
5	0.59	0.23
6a	0.30	0.50
6b	0.87	0.41
6c	0.95	0.16
7	0.43	0.53
8	0.41	0.57



**Table F3** Continued

Item/question sequence number	P-plus value	Biserial correlation
9	0.48	0.55
10	0.31	0.53
11a	0.97	0.81
11b	0.68	0.61
11c	0.98	0.62
11d	0.84	0.55
11e	0.94	0.40
12	0.33	0.35
13	0.24	0.36
14	0.50	0.65
15a	0.73	0.34
15b	0.73	0.49
15c	0.73	0.52
15d	0.62	0.58
15e	0.55	0.45
16	0.39	0.65
17	0.32	0.47
18	0.57	0.46
19	0.28	0.15
20	0.19	0.34
21	0.14	— <sup>b</sup>
22	0.56	0.32
23	0.29	0.42

<sup>a</sup>All respondents answered correctly, so no biserial correlation can be computed and item removed from final scaling.

<sup>b</sup> Item has negative or very low biserial, so is removed from final scaling.

**Table F4** English Language Arts (ELA) 4–6 Item Statistics

Item/question sequence number	P-plus value	Biserial correlation
1	0.56	0.20
2	0.64	0.46
3	0.80	0.41
4	0.34	— <sup>a</sup>
5	0.55	0.24
6a	0.69	0.38
6b	0.65	0.36
6c	0.83	0.39
6d	0.68	— <sup>a</sup>
6e	0.46	0.30
6f	0.91	0.29
7	0.51	0.42
8a	0.90	0.29
8b	0.69	0.42
8c	0.92	0.31
8d	0.40	0.35
9	0.47	0.23
10	0.34	0.46
11a	0.34	— <sup>a</sup>
11b	0.96	0.39
11c	0.89	0.51
11d	0.86	0.45
11e	0.71	0.43
11f	0.50	0.58
12a	0.52	0.30
12b	0.33	0.14

**Table F4** Continued

Item/question sequence number	P-plus value	Biserial correlation
12c	0.89	0.32
12d	0.45	— <sup>a</sup>
12e	0.64	0.20
13	0.49	0.16
14	0.92	0.40
15	0.86	0.66
16a	0.91	0.16
16b	0.86	0.11
16c	0.79	0.31
16d	0.57	0.50
16e	0.76	0.31
17	0.27	— <sup>a</sup>
18	Constructed-response item	
19a	0.82	0.25
19b	0.93	0.44
19c	0.44	0.48
19d	0.81	0.53
20	0.64	0.40
21	0.66	0.29
22	Constructed-response item	
23a	0.44	0.26
23b	0.67	0.53
23c	0.86	0.14
23d	0.80	0.22
24	0.74	0.55
25	0.54	0.37
26	0.79	0.67

<sup>a</sup>Item has negative or very low biserial, so is removed from final scaling.

**Table F5** English Language Arts (ELA) 7–9 Item Statistics

Item/question sequence number	P-plus value	Biserial correlation
1	0.20	0.27
2	0.71	0.41
3a	0.88	0.00
3b	0.11	0.11
3c	0.96	0.32
3d	0.69	0.55
4	0.31	0.18
5	0.50	0.28
6	0.52	0.47
7a	0.99	0.45
7b	0.41	0.39
7c	0.80	0.38
7d	0.92	0.42
8	0.69	0.40
9	0.68	0.15
10	0.61	0.42
11a	0.41	0.19
11b	0.94	0.18
11c	0.91	0.57
11d	0.84	0.38
11e	0.76	0.43
11f	0.62	0.59
12	Constructed-response item	

Table F5 Continued

Item/question sequence number	P-plus value	Biserial correlation
13	0.79	0.39
14	0.31	0.10
15	0.63	0.50
16a	0.69	0.47
16b	0.46	0.35
16c	0.87	0.59
17	0.67	0.48
18	Constructed-response item	
19a	0.84	0.28
19b	0.97	0.49
19c	0.52	0.45
19d	0.85	0.55
20	0.72	0.32
21a	0.70	0.08
21b	0.47	0.10
21c	0.16	— <sup>a</sup>
22	0.46	0.39
23a	0.46	0.16
23b	0.72	0.31
23c	0.87	— <sup>a</sup>
23d	0.85	0.03
24a	0.88	0.13
24b	0.75	0.48
24c	0.61	— <sup>a</sup>
24d	0.91	0.29
25	0.68	0.54
26	0.82	0.51

<sup>a</sup>Item has negative or very low biserial, so is removed from final scaling.

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